Dissertation

ANTENNA ARRAYS FOR MULTIPATH AND INTERFERENCE MITIGATION IN GNSS RECEIVERS

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OBJECTIVES

- Estimation of the time delay (and the carrier phase) of several replicas of a known signal received in a scenario with multipath propagation and/or directional interference.
- Connecting themes:
  - Application of the Maximum Likelihood Principle. Model-based estimation.
  - Noise with unknown correlation matrix.
- Applications
  - Measurement of distances: GNSS receivers, Radar, Source localization
  - Communications: Synchronization of receivers, channel identification and equalization
  - Shift between structures.

INTRODUCTION: GNSS

- Deployment in the early 70's: GPS, GLONASS.
- Constellation of satellites (~24) and several terrestrial stations that allow to obtain the tridimensional position and the time in the receiver.
- Applications: En route / precise navigation, surveying, geodesy, atmospheric study, synchronization / time transfer, fleet control, ...

  Requirements of availability, integrity and accuracy ↑↑

- Augmentation programs: LAAS, WAAS, MSAS, GNSS1 (EGNOS)
- New systems: GNSS2 (GALILEO)

- The design of improved receivers is necessary to fully exploit the capabilities of these new systems, since receiver-induced errors limit the ultimate accuracy attainable with these GNSS.

SIGNAL STRUCTURE: GPS, GLONASS

- Each satellite transmits one or several Direct Sequence – Spread Spectrum (DS–SS) signals at one or several frequencies.
- The navigation message can be modulated on some of these signals.

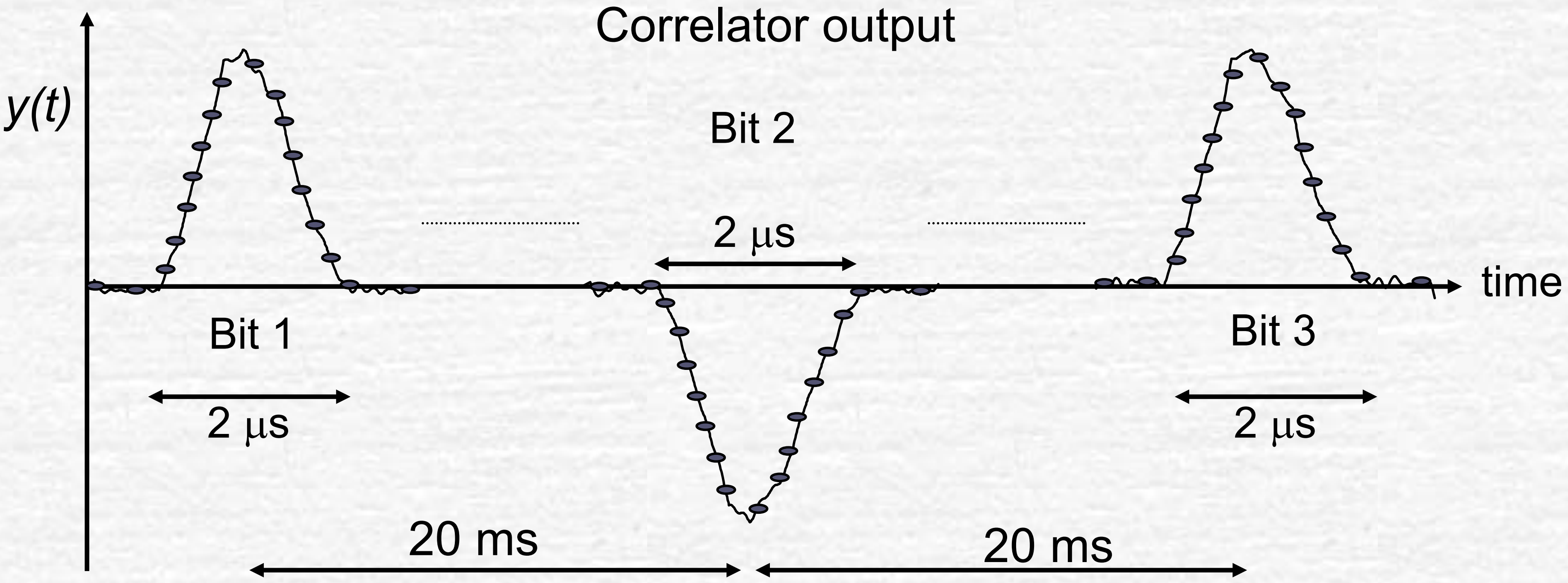
- GPS: 6 satellites in each of 4 orbital planes. Nearly circular orbits of radius 26,560 km, orbital period: 11h 58', asynchronous CDMA.
  MAI is negligible in GPS (extremely long codes and no near-far effect).
- Civilian users: L1 carrier (154×10.23MHz)
  C/A Code: Gold seq., 1.023 Mchips/s, period 1023 chips
  Navigation message: 50 bits/s
  CNo=40 – 55 dBHz
- Military users: L1 carrier and L2 carrier (120×10.23MHz)
  P Code, 10.23 Mchips/s, period of 266 days, encrypted.

- GLONASS has a very similar constellation and signal structure, but the chip rate for civil users is 511 Kchips/s and employs FDMA.

SIGNAL STRUCTURE

- SNR of the received signal ≈ -15 dB ➡ The GNSS signals are buried in the noise. The correlation matrix only contains information about the interferences. Under certain models, the array will reject the interferences but not the reflections.

- SNR after despreading ≈ 28 dB ➡ The contribution of the reflections turns up.



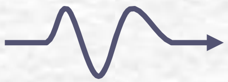
OPERATION PRINCIPLE

- Function of the receiver: Measurement of the satellite – receiver distance ➡ measurement of the propagation or time delay of the direct (line-of-sight) signal.

1) Pseudorange: Delay of the equivalent baseband signal
Non-ambiguous measurement
Standard deviation ~ 3 meters (thermal noise)

$$s_T(t) = a(t)\,e^{j2\pi f_c t}$$

i.p.c



$$s_R(t) = k\,a(t-\tau_0)\,e^{j2\pi f_c t}\,e^{-j2\pi f_c \tau_0}$$

2) Carrier Phase: Ambiguous measurement (resolution of the initial interger ambiguity and cycle slips)
Standard deviation ~ 2 mm (thermal noise)

ERROR SOURCES

- Satellite-induced errors:
  - Position (10 m ➡ 0.05 m)
  - Clock (3 m ➡ 0 m)
- Propagation-induced errors:
  - Ionosphere (15m ➡ 1 m)
  - Troposphere (2m ➡ 0.05 m)

→ Differential techniques

→ Use of two frequencies

- Receiver local errors:

  Dominant error sources in most high precision applications. Limiting factor of present GNSS.

  - Multipath (coherent) ⟶ BIAS

    Time-delay errors ~ 10, 100 meters

    Carrier-phase errors ~ centimeters

  - Interferences ⟶ VARIANCE

    Jammers are already available on the market.

CONVENTIONAL RECEPTION SCHEMES

- Received signal: $x(t) = \alpha_0 \sum_l d(l)\, p(t - lT) + w(t)$

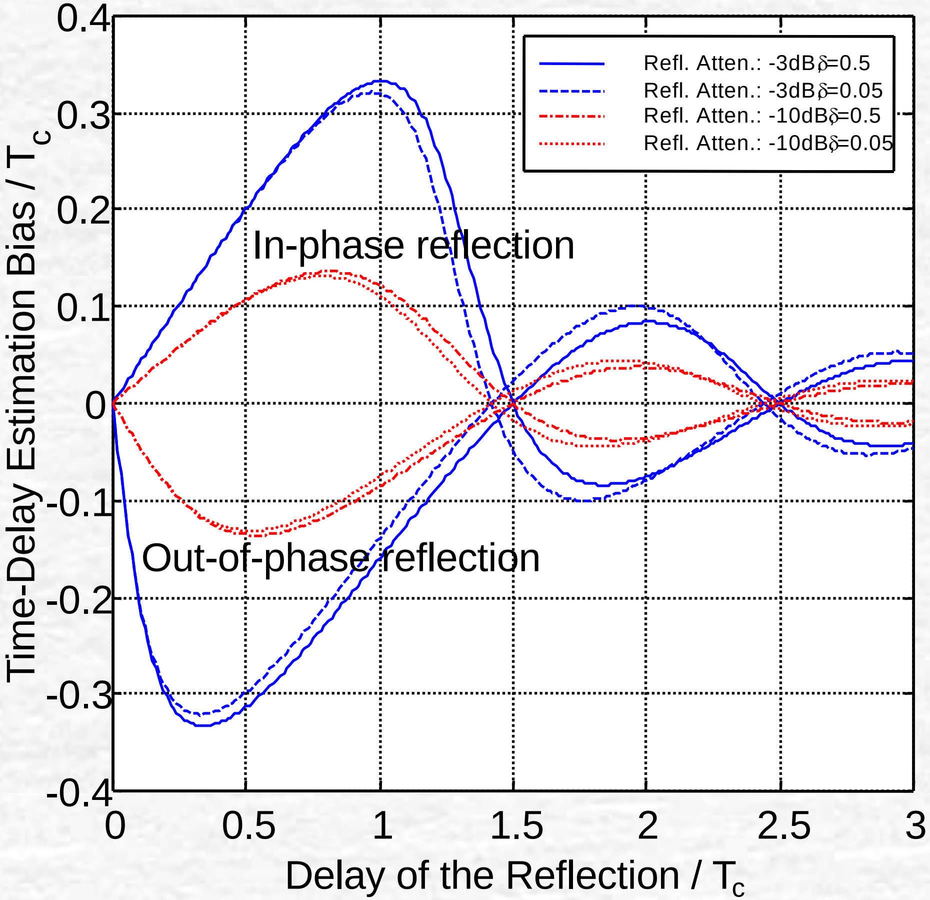
ML principle ↓ $w(t) \sim N(0, \sigma^2)$

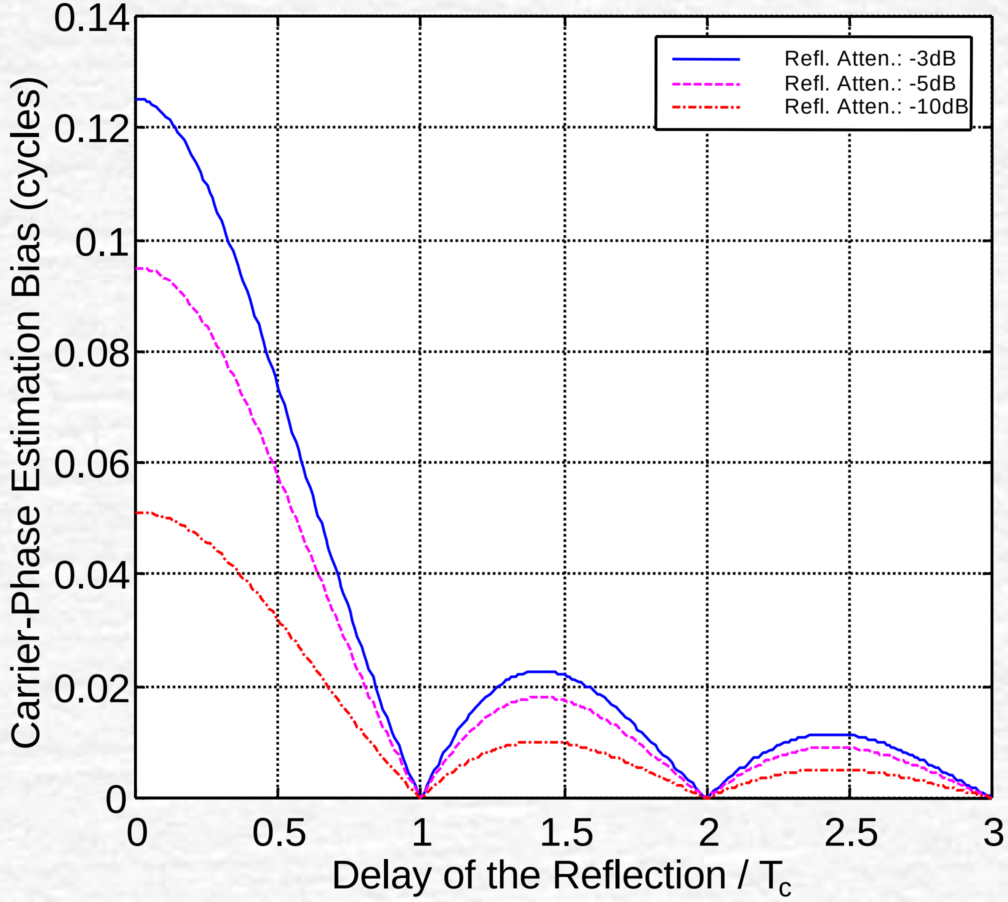
- Carrier Phase ➡ Costas loop

- Time delay ➡ DLL
  - Decision and phase directed. Coherent.
  - Non-data aided, phase independent. Non coherent.

UPC TSC

ERRORS IN CONVENTIONAL RECEIVERS





SMR = 3dB, 10dB
δ = 0.05, 0.5

SMR = 3dB, 5dB, 10dB

Square-root raised cosine pulse with 0.2 roll-off

UPC TSC

STATE-OF-THE-ART: CLASSIFICATION

- Variety of methods as a function of the application, scenario ....

- One antenna
- Antenna arrays
  - Signal processing techniques
    - Real-time, + flexibility
    - Received signal → observables
  - Post-processing techniques
    - Data processing, application-dependent
    - Observables → new observables
      final parameters

SINGLE ANTENNA METHODS

- Post-processing: changes in satellites positions ➡ periodic and slow variations of the carrier-phase and SNR. They do not combat diffuse multipath or interference.

- Real-time: correlation methods, modifications of the DLL
  - ☑ Narrow Spacing DLL (δ ~ *0.05, 0.1)*
    - ✗ reduces only the magnitude of multipath errors
    - ✗ fails with raised cosine pulses
  - ☑ Strobe Correlator™ or compensated correlators. Approximation of the derivative by more complex finite differ. Narrower S - curve
  - ☑ Edge Correlator™, e1/e2 Tracker. Location of the leading edge of the cross-correlation curve

  ✗ Noise ⬆ (Strobe Correlator™, Edge Correlator™)

  - ✗ None of these approches reduces carrier-phase errors.
  - ☑ Multipath Estimating DLL (MEDLL™): Estimator of the time-delays and amplitudes in a multipath scenario.

ANTENNA ARRAY METHODS

- Spatial filtering: special reception patterns, wise location of the antennas.

- Up to date, the use of antenna arrays in GNSS receivers has been centered on interference mitigation.
  SNRin<< 0 ➡ Output power minimization before despreading.

- DOA estimation + beamforming
  - The scenario is highly coherent ➪ decorrelation techniques or estimation methods that work with coherent sources.
  - High computational complexity (spatial searches).
  - Restrictions on the array geometry.
  - The number of signals is limited by the number of antennas.

- Processing the observables obtained at several antennas:
  - Highly non-linear problems.

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SIGNAL MODEL

- An arbitrary *m* element antenna array receives *N* samples of *d* replicas of a known signal:

$$\mathbf{Y} = \mathbf{A}\mathbf{S}(\boldsymbol{\tau}) + \mathbf{E} \qquad m \times N$$

where

$$\mathbf{A} = \begin{bmatrix} \boldsymbol{\alpha}_0 & \boldsymbol{\alpha}_1 & \cdots & \boldsymbol{\alpha}_{d-1} \end{bmatrix} \qquad m \times d$$

$$\mathbf{S}(\boldsymbol{\tau}) = \begin{bmatrix} s(T_s - \tau_0) & s(2T_s - \tau_0) & \cdots & s(NT_s - \tau_0) \\ s(T_s - \tau_1) & s(2T_s - \tau_1) & \cdots & s(NT_s - \tau_1) \\ \vdots & \vdots & \cdots & \vdots \\ s(T_s - \tau_{d-1}) & s(2T_s - \tau_{d-1}) & \cdots & s(NT_s - \tau_{d-1}) \end{bmatrix} \qquad d \times N$$

$s(t)$: finite-average-power signal. Asymptotic behaviour means $N \uparrow$

UNSTRUCTURED SPATIAL SIGNATURES NOISE MODEL

- Unstructured spatial signatures
  - Calibration of the array is not required.
  - One delay – one DOA is not required. Scattered sources can be received.
  - Reduced computational complexity, even though the number of parameters generally increases.

- Noise
  - Complex, circularly symmetric Gaussian vector.
  - Temporally white.
  - Unknonw spatial correlation matrix ➩ Robustness against co-channel interference.

$$\mathrm{E}\left\{\mathbf{e}[n]\mathbf{e}^{*}[l]\right\} = \mathbf{Q}\,\delta_{n,l}$$

  - Trade-off between model realism and complexity.

MAXIMUM LIKELIHOOD ESTIMATOR (1/2)

- Negative log-likelihood

$$f(\boldsymbol{\tau}, \mathbf{A}, \mathbf{Q}) = \ln|\mathbf{Q}| + \mathrm{Tr}\left\{\mathbf{C}(\boldsymbol{\tau}, \mathbf{A})\mathbf{Q}^{-1}\right\}$$

"Regularization" term   Known **Q**

$$\mathbf{C}(\boldsymbol{\tau}, \mathbf{A}) = \hat{\mathbf{R}}_{yy} - \mathbf{A}\hat{\mathbf{R}}_{ys}^{*}(\boldsymbol{\tau}) - \hat{\mathbf{R}}_{ys}(\boldsymbol{\tau})\mathbf{A}^{*} + \mathbf{A}\hat{\mathbf{R}}_{ss}(\boldsymbol{\tau})\mathbf{A}^{*}$$

- After optimization with respect to **Q**

$$f(\boldsymbol{\tau}, \mathbf{A}) = \ln\Big|\hat{\mathbf{R}}_{yy} - \hat{\mathbf{R}}_{ys}(\boldsymbol{\tau})\hat{\mathbf{R}}_{ss}^{-1}(\boldsymbol{\tau})\hat{\mathbf{R}}_{ys}^{*}(\boldsymbol{\tau})$$
$$+\left(\mathbf{A} - \hat{\mathbf{R}}_{ys}(\boldsymbol{\tau})\hat{\mathbf{R}}_{ss}^{-1}(\boldsymbol{\tau})\right)\hat{\mathbf{R}}_{ss}(\boldsymbol{\tau})\left(\mathbf{A} - \hat{\mathbf{R}}_{ys}(\boldsymbol{\tau})\hat{\mathbf{R}}_{ss}^{-1}(\boldsymbol{\tau})\right)^{*}\Big|$$

MAXIMUM LIKELIHOOD ESTIMATOR (2/2)

- After optimization with respect to **A**

$$f(\boldsymbol{\tau}) = \ln\left|\hat{\mathbf{R}}_{yy} - \hat{\mathbf{R}}_{ys}(\boldsymbol{\tau})\hat{\mathbf{R}}_{ss}^{-1}(\boldsymbol{\tau})\hat{\mathbf{R}}_{ys}^{*}(\boldsymbol{\tau})\right| = \ln\left|\hat{\mathbf{Q}}_{ML}(\boldsymbol{\tau})\right|$$

It is a measure of the "magnitude" of the correlation matrix of the residuals after a least square fit. This measure is the geometric mean of the eigenvalues of $\hat{\mathbf{Q}}_{ML}(\boldsymbol{\tau})$.

- An equivalent expression is

$$V(\boldsymbol{\tau}) = \ln\left|\mathbf{I} - \mathbf{B}(\boldsymbol{\tau})\right|$$

where $\mathbf{B}(\boldsymbol{\tau}) = \hat{\mathbf{R}}_{yy}^{-1/2}\hat{\mathbf{R}}_{ys}(\boldsymbol{\tau})\hat{\mathbf{R}}_{ss}^{-1}(\boldsymbol{\tau})\hat{\mathbf{R}}_{ys}^{*}(\boldsymbol{\tau})\hat{\mathbf{R}}_{yy}^{-1/2} = \frac{1}{N}\hat{\mathbf{R}}_{yy}^{-1/2}\mathbf{Y}\mathbf{P}_{\mathbf{S}^{*}(\boldsymbol{\tau})}\mathbf{Y}^{*}\hat{\mathbf{R}}_{yy}^{-1/2}$

  - It is consistent and asymptotically efficient,
  - but highly non-linear due to the determinant ⇨ multidimensional search.

SPATIALLY WHITE NOISE

- If the noise is assumed to be spatially white

$$f^{w}(\boldsymbol{\tau}) = \mathrm{Tr}\left\{\hat{\mathbf{Q}}_{ML}(\boldsymbol{\tau})\right\} = \text{const-Tr}\left\{\mathbf{Y}\mathbf{P}_{\mathbf{S}^*(\boldsymbol{\tau})}\mathbf{Y}^*\right\}$$

In this case, the measure of the magnitude of $\hat{\mathbf{Q}}_{ML}(\boldsymbol{\tau})$ is the arithmetic mean of its eigenvalues.

- Linear dependence on $\mathbf{P}_{\mathbf{S}^*(\boldsymbol{\tau})}$ ➡ the IQML (and ESPRIT) algorithms can be applied.

- Frequency-domain representation

$$\mathbf{S}^*(\boldsymbol{\tau}) = \mathbf{S}^*_{\omega}\begin{bmatrix} e^{j\omega_1\tau_0} & \cdots & e^{j\omega_1\tau_{d-1}} \\ e^{j\omega_2\tau_0} & \cdots & e^{j\omega_2\tau_{d-1}} \\ \vdots & & \vdots \\ e^{j\omega_N\tau_0} & & e^{j\omega_N\tau_{d-1}} \end{bmatrix}$$

ASYMPTOTICALLY EQUIVALENT ESTIMATOR

- Objective: To obtain a cost function for the correlated-noise case that is linear in the signal projection matrix.
  The following criterion is asymptotically equivalent to the ML one.

$$g(\boldsymbol{\tau}, \mathbf{W}_{\mathrm{op}}) = -\mathrm{Tr}\left\{ \mathbf{W}_{\mathrm{op}} \mathbf{B}(\boldsymbol{\tau}) \right\} \qquad \text{where} \qquad \mathbf{W}_{\mathrm{op}} = \left(\mathbf{I} - \mathbf{B}(\breve{\boldsymbol{\tau}}) \right)^{-1}$$

  since

$$g^{(i)}(\breve{\boldsymbol{\tau}}, \mathbf{W}_{\mathrm{op}}) = V^{(i)}(\breve{\boldsymbol{\tau}}) + o_p(N^{-1/2})$$

$$g^{(ij)}(\breve{\boldsymbol{\tau}}, \mathbf{W}_{\mathrm{op}}) = V^{(ij)}(\breve{\boldsymbol{\tau}}) + o_p(1)$$

- Alternative: Computing the matrix **W** that minimizes the error covariance.

- The optimum weigthing matrix can be replaced by a consistent estimate. The consistent estimate of the delays $\hat{\boldsymbol{\tau}}$ is obtained by minimizing $g(\boldsymbol{\tau}, \mathbf{I})$, which performs much better than $f^{w}(\boldsymbol{\tau})$.

HEURISTIC DERIVATIONS (1/3)

- They shed light on how to approximate the determinant.
- Series expansion of the ML cost function

$$V(\boldsymbol{\tau}) = \ln\left|\mathbf{I} - \mathbf{B}(\boldsymbol{\tau})\right| = -\mathrm{Tr}\left\{\mathbf{B}(\boldsymbol{\tau})\right\} - \frac{1}{2}\mathrm{Tr}\left\{\mathbf{B}^2(\boldsymbol{\tau})\right\} - \frac{1}{3}\mathrm{Tr}\left\{\mathbf{B}^3(\boldsymbol{\tau})\right\} - \cdots$$

  - Unlike many other estimation problems, the first-order term is not asymptotically equivalent to the original function because

$$\lim_{N\to\infty} \mathbf{B}(\boldsymbol{\tau}) = \mathbf{I} - \mathbf{R}_{yy}^{-1/2}\mathbf{Q}\mathbf{R}_{yy}^{-1/2} \neq \mathbf{0}$$

  - Therefore, we retain and approximate the second- and higher order terms, which are the ones that introduce the undesired non-linear dependence on the signal projection matrix:

$$V^{(i)}(\boldsymbol{\tau}) \approx -\mathrm{Tr}\left\{\left(\mathbf{I} + \mathbf{B}(\hat{\boldsymbol{\tau}}) + \mathbf{B}^2(\hat{\boldsymbol{\tau}}) + \cdots\right)\mathbf{B}^{(i)}(\boldsymbol{\tau})\right\} = g^{(i)}(\boldsymbol{\tau}, \hat{\mathbf{W}})$$

HEURISTIC DERIVATIONS (2/3)

- Eigenvalue weigthing
  - The geometric mean of the eigenvalues can be view as a weigthed arithmetic mean.
    - ML cost function and asymptotically equivalent approximation

$$V(\boldsymbol{\tau}) = \ln\left|\mathbf{I} - \mathbf{B}(\boldsymbol{\tau})\right| = \ln \prod_{k=1}^{m}\left(1 - \lambda_k(\boldsymbol{\tau})\right)$$

$$V^{(i)}(\boldsymbol{\tau}) = -\sum_{k=1}^{m} \frac{\lambda_k^{(i)}(\boldsymbol{\tau})}{1 - \lambda_k(\boldsymbol{\tau})} \approx -\sum_{k=1}^{m} \frac{\lambda_k^{(i)}(\boldsymbol{\tau})}{1 - \lambda_k(\hat{\boldsymbol{\tau}})} \approx g^{(i)}(\boldsymbol{\tau}, \hat{\mathbf{W}})$$

    - Cost function providing the consistent estimates

$$g(\boldsymbol{\tau}, \mathbf{I}) = -\mathrm{Tr}\left\{\mathbf{B}(\boldsymbol{\tau})\right\} = -\sum_{k=1}^{m} \lambda_k(\boldsymbol{\tau}) \quad \Rightarrow \quad g^{(i)}(\boldsymbol{\tau}, \mathbf{I}) = -\sum_{k=1}^{m} \lambda_k^{(i)}(\boldsymbol{\tau})$$

- The difference between them is an appropriate weighting of the eigenvalues.

HEURISTIC DERIVATIONS (3/3)

- Modified first order approximation

  - Using a simple trick, a first-order approximation of the ML criterion can be asymptotically efficient.

$$V(\boldsymbol{\tau}) = \ln\left|\mathbf{I} - \mathbf{B}(\hat{\boldsymbol{\tau}}) + \mathbf{B}(\hat{\boldsymbol{\tau}}) - \mathbf{B}(\boldsymbol{\tau})\right|$$

$$= \ln\left|\mathbf{I} - \mathbf{B}(\hat{\boldsymbol{\tau}})\right| + \ln\left|\mathbf{I} + \left(\mathbf{I} - \mathbf{B}(\hat{\boldsymbol{\tau}})\right)^{-1}\left(\mathbf{B}(\hat{\boldsymbol{\tau}}) - \mathbf{B}(\boldsymbol{\tau})\right)\right|$$

$\hat{\mathbf{W}}$

and approximating the determinant by the trace

$$V(\boldsymbol{\tau}) \approx \ln\left|\mathbf{I} - \mathbf{B}(\hat{\boldsymbol{\tau}})\right| + \mathrm{Tr}\left\{\hat{\mathbf{W}}\mathbf{B}(\hat{\boldsymbol{\tau}})\right\} - \mathrm{Tr}\left\{\hat{\mathbf{W}}\mathbf{B}(\boldsymbol{\tau})\right\} = \mathrm{const} + g(\boldsymbol{\tau}, \hat{\mathbf{W}})$$

APPLICATION OF IQML

$$g(\boldsymbol{\tau}, \mathbf{W}) = -\frac{1}{N}\mathrm{Tr}\left\{\mathbf{W}^{1/2}\hat{\mathbf{R}}_{\acute{y}y}^{-1/2}\mathbf{Y}\mathbf{P}_{\mathbf{S}^*(\boldsymbol{\tau})}\mathbf{Y}^*\hat{\mathbf{R}}_{\acute{y}y}^{-1/2}\mathbf{W}^{1/2}\right\}$$

- Parameterization of the signal projection matrix in the frequency domain

$$\mathbf{P}_{\mathbf{S}^*(\boldsymbol{\tau})}^{\perp} = \mathbf{P}_{\mathbf{S}_\omega^{-1}\mathbf{G}} = \mathbf{S}_\omega^{-1}\mathbf{G}\left(\mathbf{G}^*\mathbf{S}_\omega^{-*}\mathbf{S}_\omega^{-1}\mathbf{G}\right)^{-1}\mathbf{G}^*\mathbf{S}_\omega^{-*}$$

The dependence on the elements of **G** is quadratic if the central inverse matrix is held fixed.

<u>2 iterative processes</u>

- IQML
- Computation of **W**

⇨

A) Coupled iterations

B) Decoupled iterations

SIMULATION PARAMETERS

- The signal consists of $M=3$ square root raised cosine pulses with r/o 0.2
- Each pulse is truncated to $\pm 3T_c$, sampling frequency $T_c/2$
- ULA, $m=6$ antennas, 0.5λ
- $d=2$ rays
  - delays: $0T_c$, $0.4T_c$
  - DOAs: 0º, 10º.
  - SNR_0=16dB. Attenuation of the 2^{nd} ray: 3dB
- One interferer
  - DOA: -30º
  - SIR_0= -3dB

- Initialization of IQML using LS-ESPRIT with δ=2.
- IQML is implemented with a quadratic contraint.

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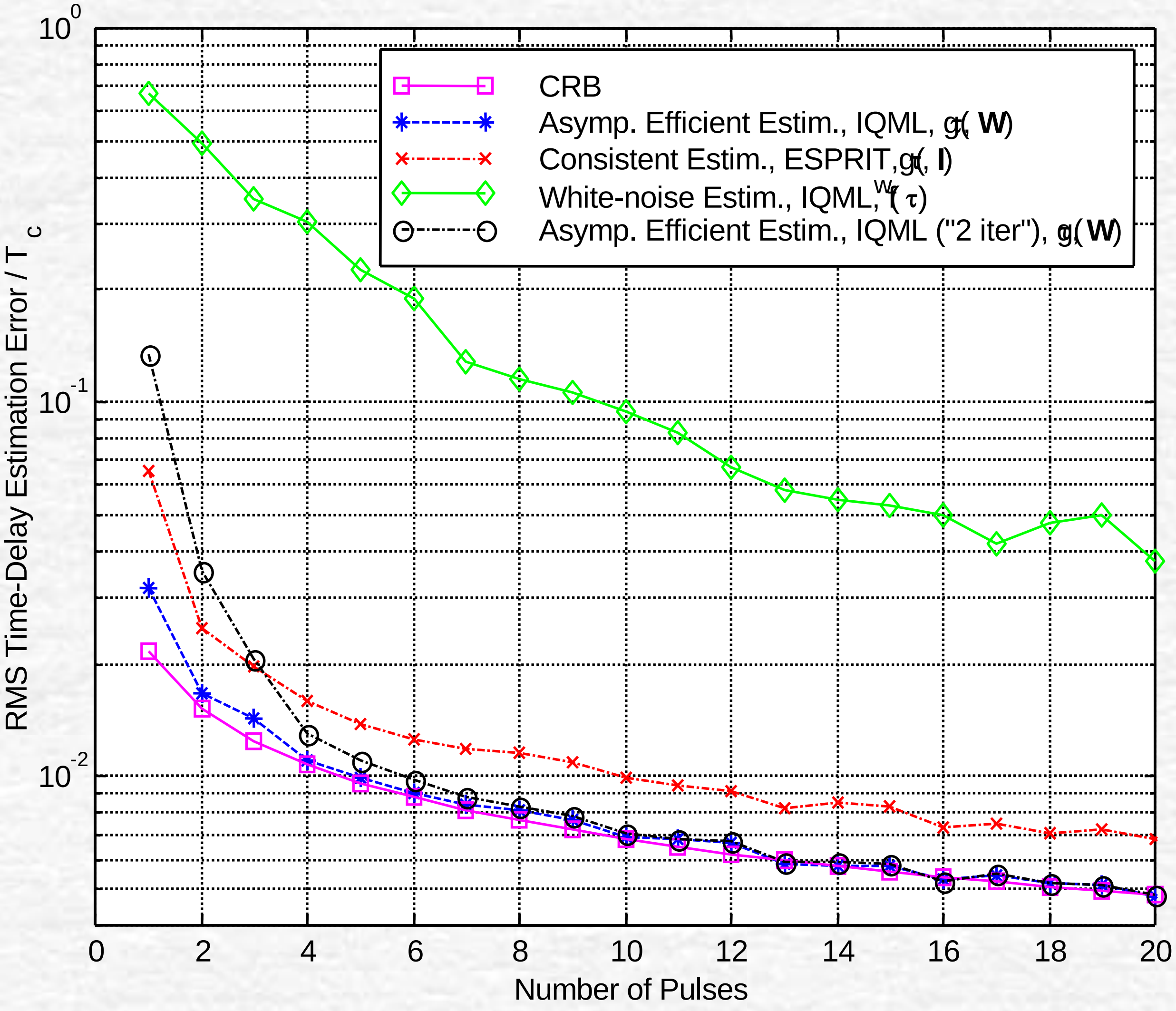
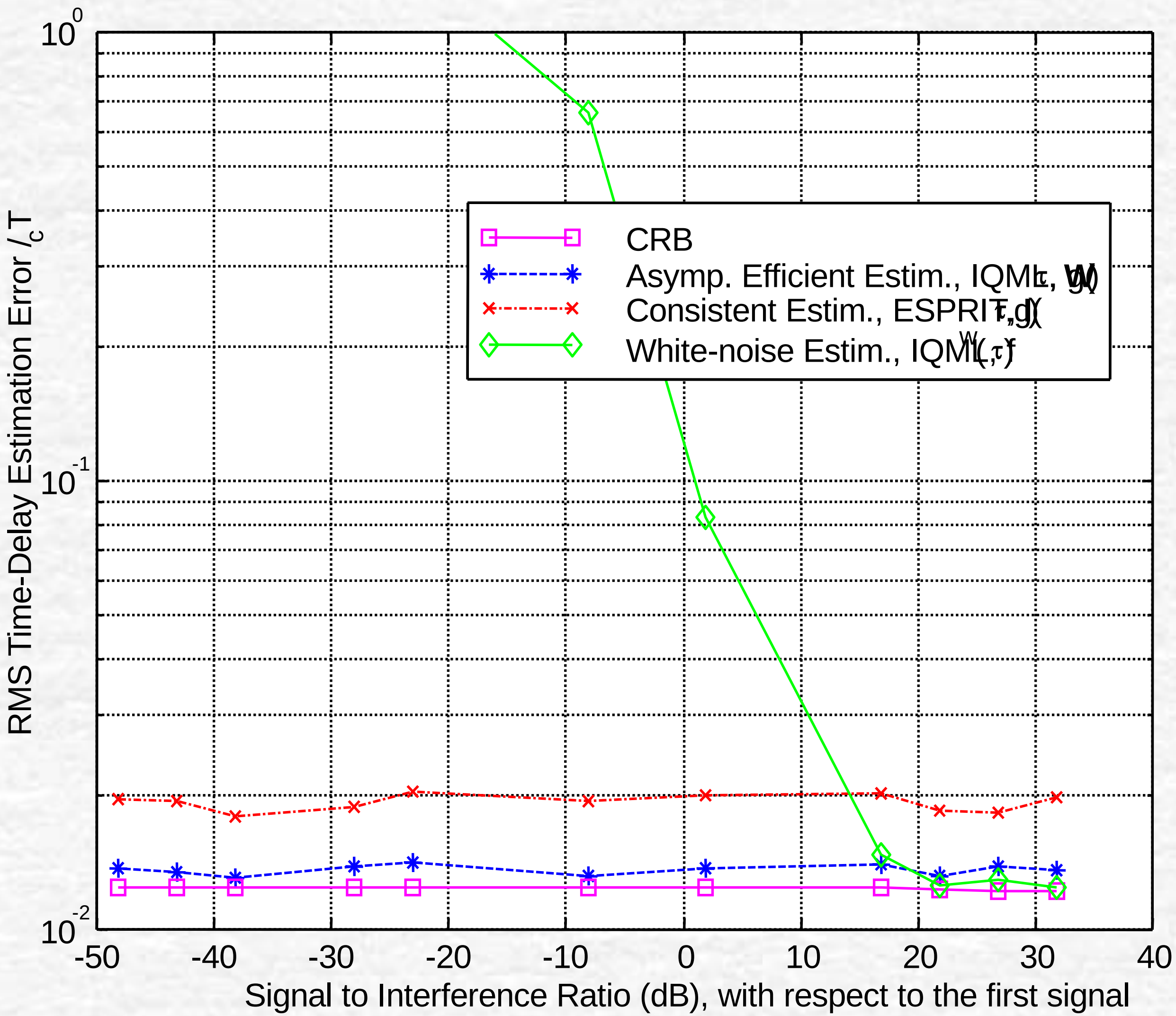
SIMULATION RESULTS (1/4)



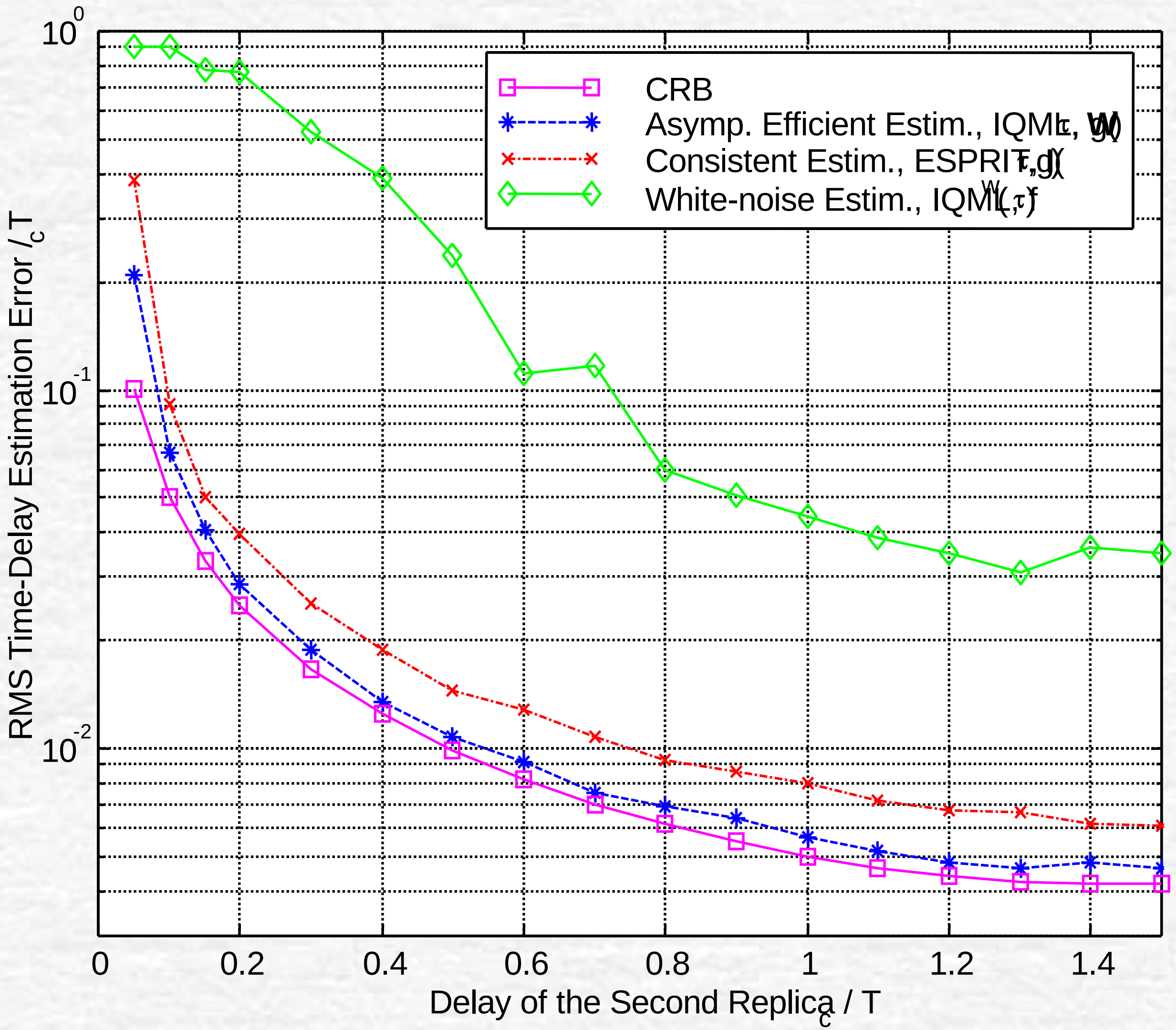
- The proposed estimators attain the CRB. The coupled iterations' approach has advantages (smaller RMSE and number of iterations).
- The consistent estimator is not optimum but is robust against directional interferers. It takes the spatial correlation into account but not in an optimal way.

SIMULATION RESULTS (2/4)



- The method derived under the white-noise assumption is not suited for strong CCI, though it employs antenna arrays.
- The consistent and asymptotically efficient estimators are nearly insensitive to the interference power.

SIMULATION RESULTS (3/4)



- The RMSE and the CRB increase as the delay separation decreases.

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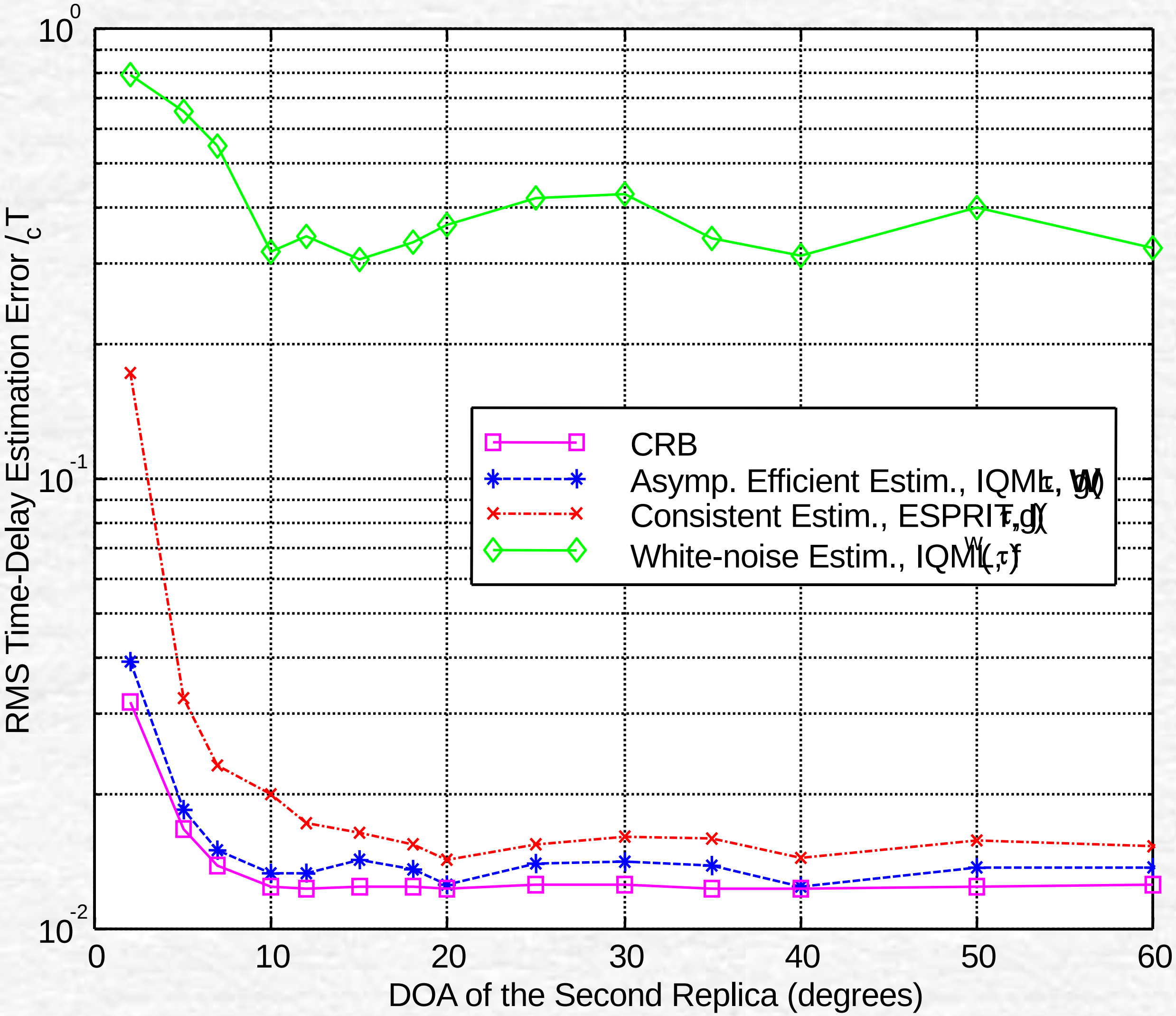
SIMULATION RESULTS (4/4)



- The CRB increases when the DOA separation is smaller than the array beamwidth (but it does not tend to infinity).
- For close DOAs, the ESPRIT is deteriorated because **A** tends to be rank-deficient. However, the proposed estimator remains close to the CRB.

MODIFIED IQML FOR FIR CHANNELS (1/2)

- FIR channel $\mathbf{S}(\tau) = \left[\mathbf{s}(\tau) \mid \mathbf{s}(\tau + T_0) \mid \cdots \mid \mathbf{s}(\tau + (d-1)T_0)\right]^T$

- Modification of IQML ➪ Polynomial rooting
  - Cost function: $g(\tau, \mathbf{W}) = \dfrac{1}{N}\mathrm{Tr}\left\{\mathbf{W}^{1/2}\hat{\mathbf{R}}_{yy}^{-1/2}\mathbf{Y}\mathbf{P}^{\perp}_{\mathbf{S}^*(\tau)}\mathbf{Y}^*\hat{\mathbf{R}}_{yy}^{-1/2}\mathbf{W}^{1/2}\right\}$
  - Parameterization: $\mathbf{P}^{\perp}_{\mathbf{S}^*(\tau)} = \mathbf{P}_{\mathbf{S}_\omega^{-1}\mathbf{G}} = \mathbf{S}_\omega^{-1}\mathbf{G}\left(\mathbf{G}^*\mathbf{S}_\omega^{-*}\mathbf{S}_\omega^{-1}\mathbf{G}\right)^{-1}\mathbf{G}^*\mathbf{S}_\omega^{-*}$

$$\mathbf{G}^* = \begin{bmatrix} g_d & g_{d-1} & \cdots & 1 & 0 & & \\ 0 & g_d & g_{d-1} & \cdots & 1 & 0 & \\ & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\ & & 0 & g_d & g_{d-1} & \cdots & 1 \end{bmatrix}$$

where $\mathbf{g} = \left[1 \quad g_1 \quad \cdots \quad g_d\right]^T$ are the coefficients of the polynomial $G(z)$.

MODIFIED IQML FOR FIR CHANNELS (2/2)

$$G(z) = z^d + g_1 z^{d-1} + \cdots + g_d = \prod_{n=0}^{d-1} \left(z - x \cdot \exp(j2\pi T_0 n / NT_s) \right)$$

where $x(\tau) = \exp(j2\pi\tau / NT_s)$.

- Given the common factor *x* in the roots of $G(z)$, the coefficients satisfy

$$\boxed{\mathbf{g} = \mathbf{K}\mathbf{t}(x)}$$ where **K** is a known diagonal matrix and

$$\mathbf{t}(x) = \begin{bmatrix} 1 & x & \cdots & x^d \end{bmatrix}^T .$$

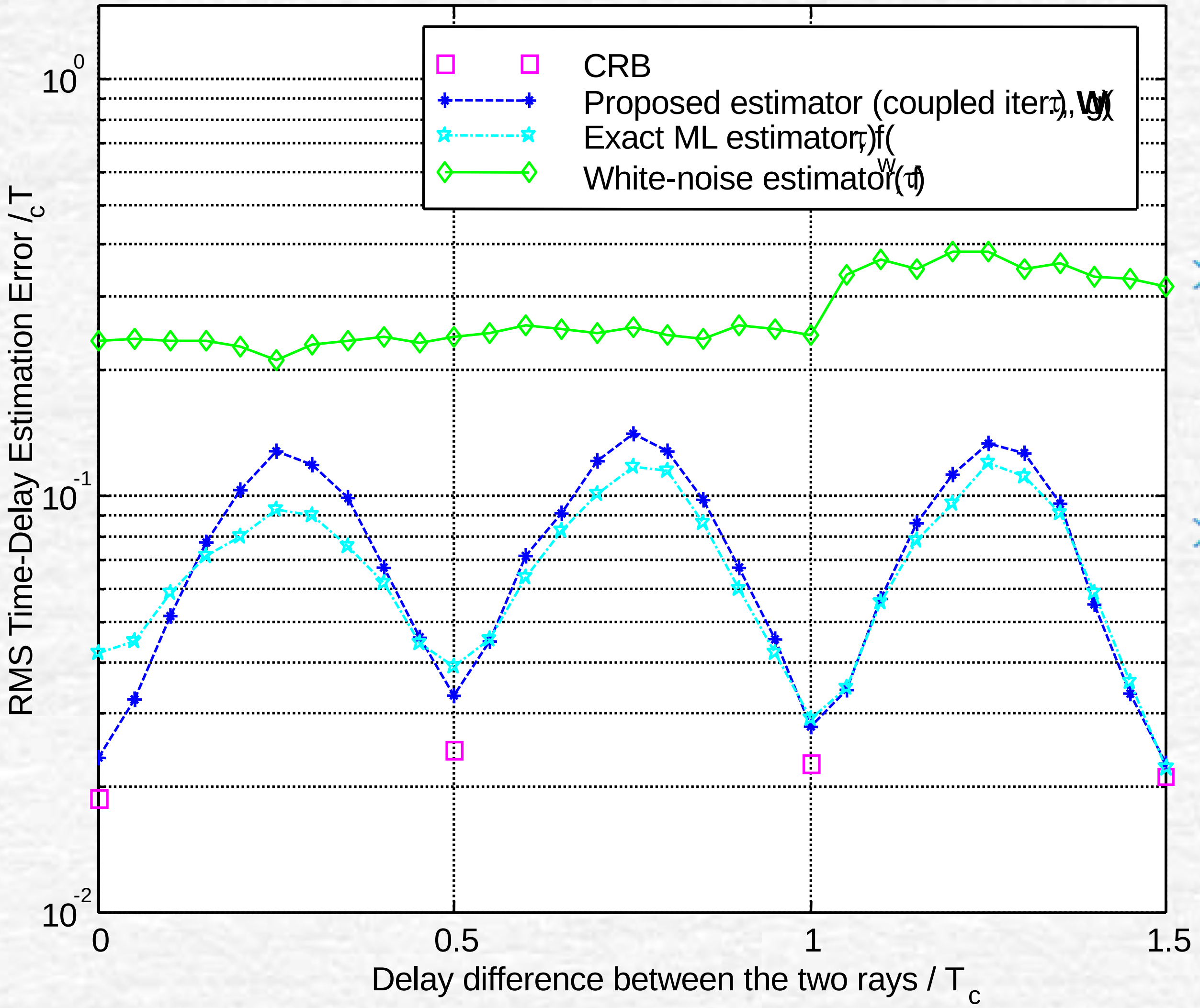
- If the term $\left(\mathbf{G}^* \mathbf{S}_\omega^{-*} \mathbf{S}_\omega^{-1} \mathbf{G} \right)^{-1}$ is computed using a previous estimate of *x*, then the cost function becomes quadratic in the coefficients **g**, so

$$g(x, \mathbf{W}) = \mathbf{t}^T (1/x) \mathbf{K}^* \mathbf{C} \mathbf{K} \mathbf{t}(x)$$

is a polynomial of order *2d* in *x*.

- The update of $\left(\mathbf{G}^* \mathbf{S}_\omega^{-*} \mathbf{S}_\omega^{-1} \mathbf{G} \right)^{-1}$ does not involve the inversion of a matrix.

SIMULATION RESULTS



- Model of the FIR channel:
  - 4 taps
  - T_0= 0.5 T_c
- Received signals
  - 2 rays
  - The delay spacing $\tau_1 - \tau_0$ is varied.

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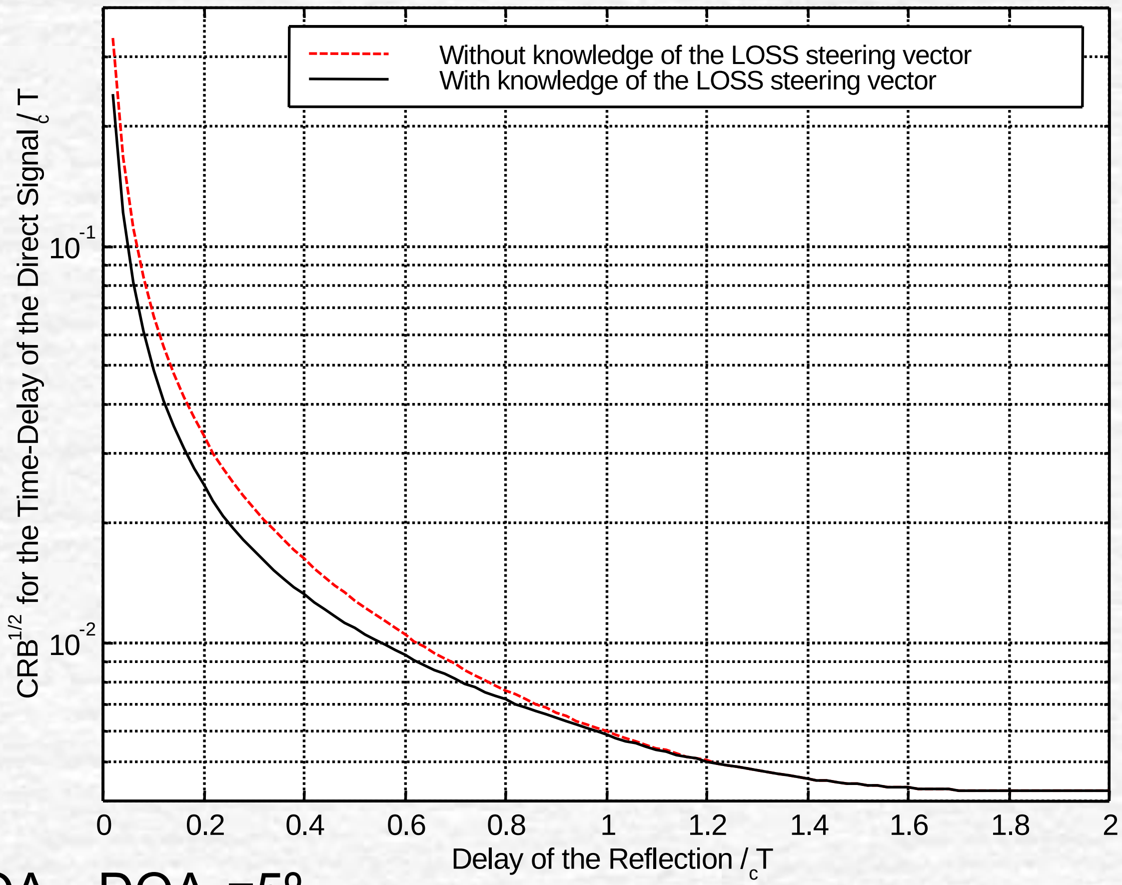
KNOWN STEERING VECTOR OF THE DIRECT SIGNAL

- The spatial signature of the line-of-sight signal is known up to a complex scaling constant

$$\boldsymbol{\alpha}_0 = \alpha_0 \mathbf{a}_0$$



DOA$_1$- DOA$_0$=5º

1) Insignificant performance improvement.

2) Increased computational load.

⬇

Simplification of the signal model.

MAXIMUM LIKELIHOOD ESTIMATOR (1/2)

- Signal model $$\mathbf{Y} = \alpha_0 \mathbf{a}_0 \mathbf{s}^T(\tau_0) + \mathbf{E}$$

- Inverse likelihood function (concentrated with respect to **Q** and α_0)

$$\Gamma(\tau_0) = \underbrace{\left|\hat{\mathbf{W}}(\tau_0)\right|}_{\text{Unstructured spatial signature}} \cdot \underbrace{\left(1 + \hat{P}_s^{-1}\hat{\mathbf{r}}_{ys}^*(\tau_0)\hat{\mathbf{W}}^{-1}(\tau_0)\hat{\mathbf{r}}_{ys}^*(\tau_0) - \frac{\left|\hat{\mathbf{r}}_{ys}^*(\tau_0)\hat{\mathbf{W}}^{-1}(\tau_0)\mathbf{a}_0\right|^2}{\hat{P}_s \mathbf{a}_0^H \hat{\mathbf{W}}^{-1}(\tau_0)\mathbf{a}_0}\right)}_{\text{Knowledge of } \mathbf{a}_0}$$

Unstructured spatial signature

Knowledge of $\mathbf{a}_0$

$$\hat{\mathbf{W}}(\tau_0) = \hat{\mathbf{R}}_{yy} - \hat{\mathbf{r}}_{ys}(\tau_0)\hat{\mathbf{r}}_{ys}^*(\tau_0)\hat{P}_s^{-1}$$

Unstructured estimate of the noise plus interference correlation.

MAXIMUM LIKELIHOOD ESTIMATOR (2/2)

- Alternative expression of the inverse likelihood

$$\Gamma(\tau_0) = \left|\hat{\mathbf{R}}_{yy}\right| \cdot \left(1 - \hat{\alpha}_{0,ML}(\tau_0)\hat{\mathbf{r}}_{ys}^*(\tau_0)\hat{\mathbf{R}}_{yy}^{-1}\mathbf{a}_0\right)$$

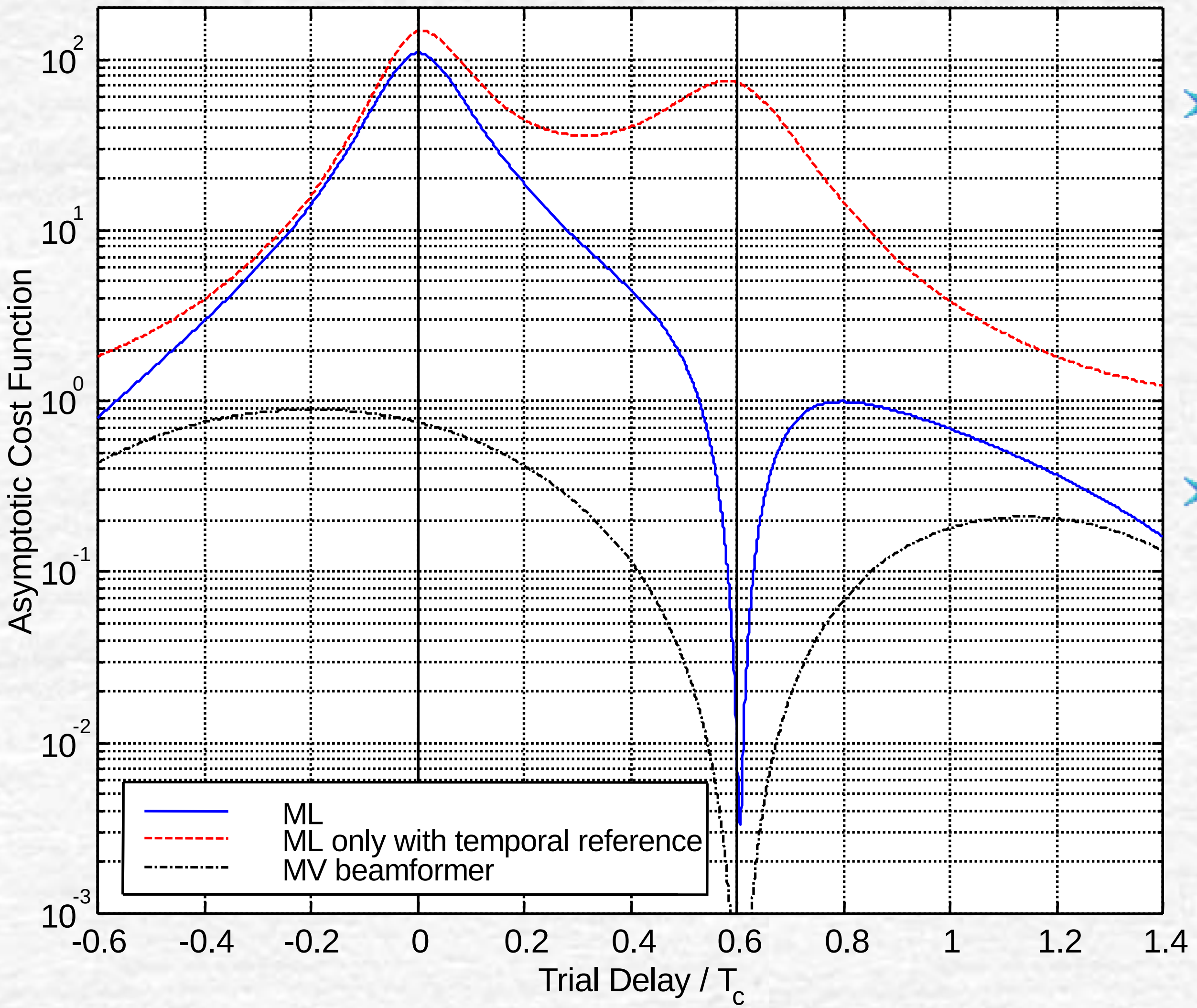
- Amplitude estimate

$$\hat{\alpha}_{0,ML} = \left.\frac{\mathbf{a}_0^*\hat{\mathbf{W}}^{-1}(\tau_0)\hat{\mathbf{r}}_{ys}(\tau_0)}{\hat{P}_s\mathbf{a}_0^*\hat{\mathbf{W}}^{-1}(\tau_0)\mathbf{a}_0}\right|_{\tau_0=\hat{\tau}_{0,ML}}$$

- Time delay estimate

$$\hat{\tau}_{0,ML} \rightarrow \Lambda_{ML}(\tau_0) = \frac{\left|\mathbf{a}_0^*\hat{\mathbf{R}}_{yy}^{-1}\hat{\mathbf{r}}_{ys}(\tau_0)\right|^2}{\hat{P}_s - \hat{\mathbf{r}}_{ys}^*(\tau_0)\hat{\mathbf{R}}_{yy}^{-1}\hat{\mathbf{r}}_{ys}(\tau_0)} = \frac{\Lambda_{MV}(\tau_0)}{1-\Lambda_{TE}(\tau_0)}$$

RELATION WITH OTHER ESTIMATORS



- ML with only temporal ref.
  - presents a peak for each reflection.
  - roughly determines the position of the peak
- MV beamformer
  - presents nulls for all reflections, but not for the direct signal.
  - determines which peak corresponds to the direct signal.

CLOSED-FORM SOLUTIONS

- Piecewise linear dependence

$$\mathbf{s}(\tau_0) \approx (1-\delta)\mathbf{s}\left(pT_0\right) + \delta\mathbf{s}\left((p+1)T_0\right)$$ in the *p*-th interval

⬇

$\Lambda_{ML}(\tau_0)$ becomes a quotient of second-order polynomial in δ.

- Frequency domain

$$\mathbf{s}(\tau_0) = \mathbf{S}_\omega \mathbf{u}(z)\Big|_{z=\exp(-j2\pi\tau_0/NT_s)}$$ where $\mathbf{u}(z) = \begin{bmatrix} 1 & z & \quad & z^{N-1} \end{bmatrix}^T$

$$\Lambda_{ML}(\tau_0) = \frac{\mathbf{u}(z^{-1})\mathbf{S}_\omega \mathbf{Y}^* \hat{\mathbf{R}}_{yy}^{-1} \mathbf{a}_0 \mathbf{a}_0^* \hat{\mathbf{R}}_{yy}^{-1} \mathbf{Y} \mathbf{S}_\omega^* \mathbf{u}(z)}{\mathbf{u}(z^{-1})\mathbf{S}_\omega \left(N\mathbf{I} - \mathbf{Y}^* \hat{\mathbf{R}}_{yy}^{-1} \mathbf{Y}\right) \mathbf{S}_\omega^* \mathbf{u}(z)}$$

Quotient of (*2N-2*)th-order polynomials.

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ITERATIVE SOLUTION: HYBRID BEAMFORMING (1/2)

- The minimization of the MSE between the output of a beamformer and a partially known reference signal

$$\begin{cases} \hat{\tau}_0, \hat{\alpha}_0, \mathbf{w}_{hyb} = \arg \min\limits_{\tau_0, \alpha_0, \mathbf{w}} \left\| \mathbf{w}^* \mathbf{Y} - \alpha_0 \mathbf{s}^T(\tau_0) \right\|_2^2 \\ \text{subject to the spatial constraint} \quad \mathbf{w}^* \mathbf{a}_0 = 1 \end{cases}$$

yields the same estimates of τ_0, α_0 as the ML approach.

- In order to obtain an iterative implementation, the optimum beamformer is computed for fixed τ_0, α_0.

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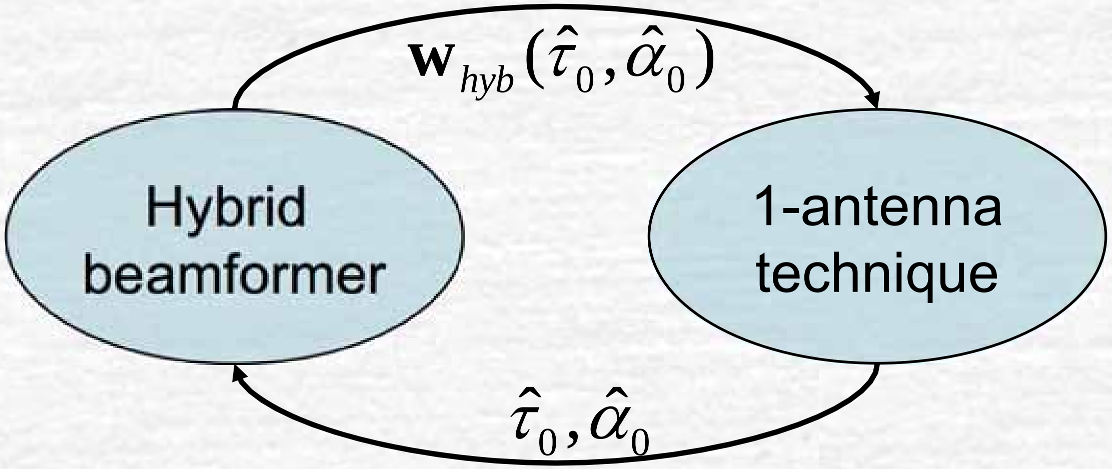
ITERATIVE SOLUTION: HYBRID BEAMFORMING (2/2)

$$\mathbf{w}_{hyb}(\tau_0,\alpha_0) = \alpha_0^* \underbrace{\hat{\mathbf{R}}_{yy}^{-1}\mathbf{r}_{ys}(\tau_0)}_{\text{temporal reference}} + \beta(\tau_0,\alpha_0)\underbrace{\frac{\hat{\mathbf{R}}_{yy}^{-1}\mathbf{a}_0}{\mathbf{a}_0^*\hat{\mathbf{R}}_{yy}^{-1}\mathbf{a}_0}}_{\text{spatial reference}}$$

temporal reference: combines the reflections in phase with the direct signal

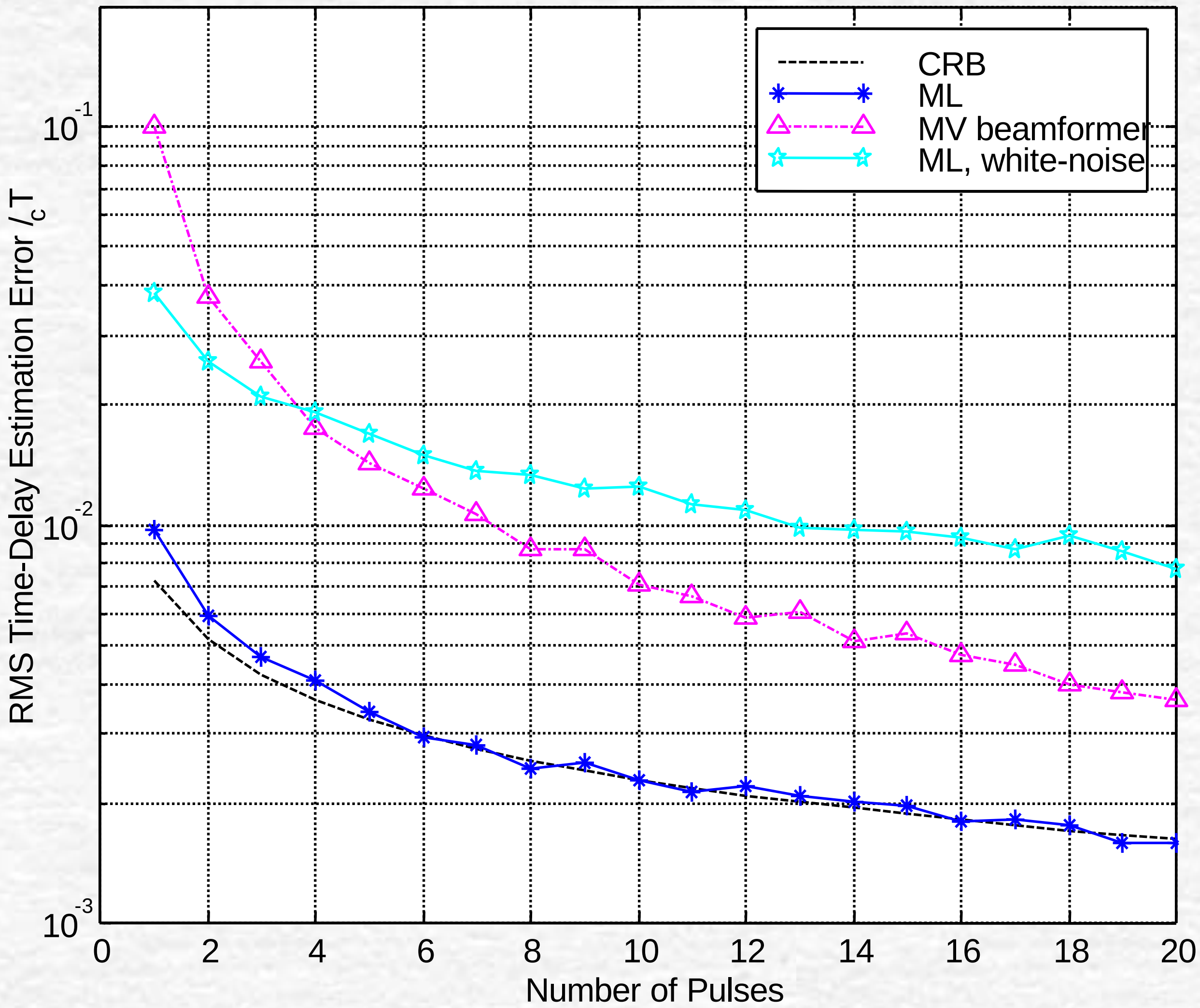
spatial reference: combines the reflections in counter-phase with the direct signal

$$\beta(\tau_0,\alpha_0) = 1 - \alpha_0^*\mathbf{a}_0^*\hat{\mathbf{R}}_{yy}^{-1}\mathbf{r}_{ys}(\tau_0)$$



SIMULATION RESULTS (1/7)

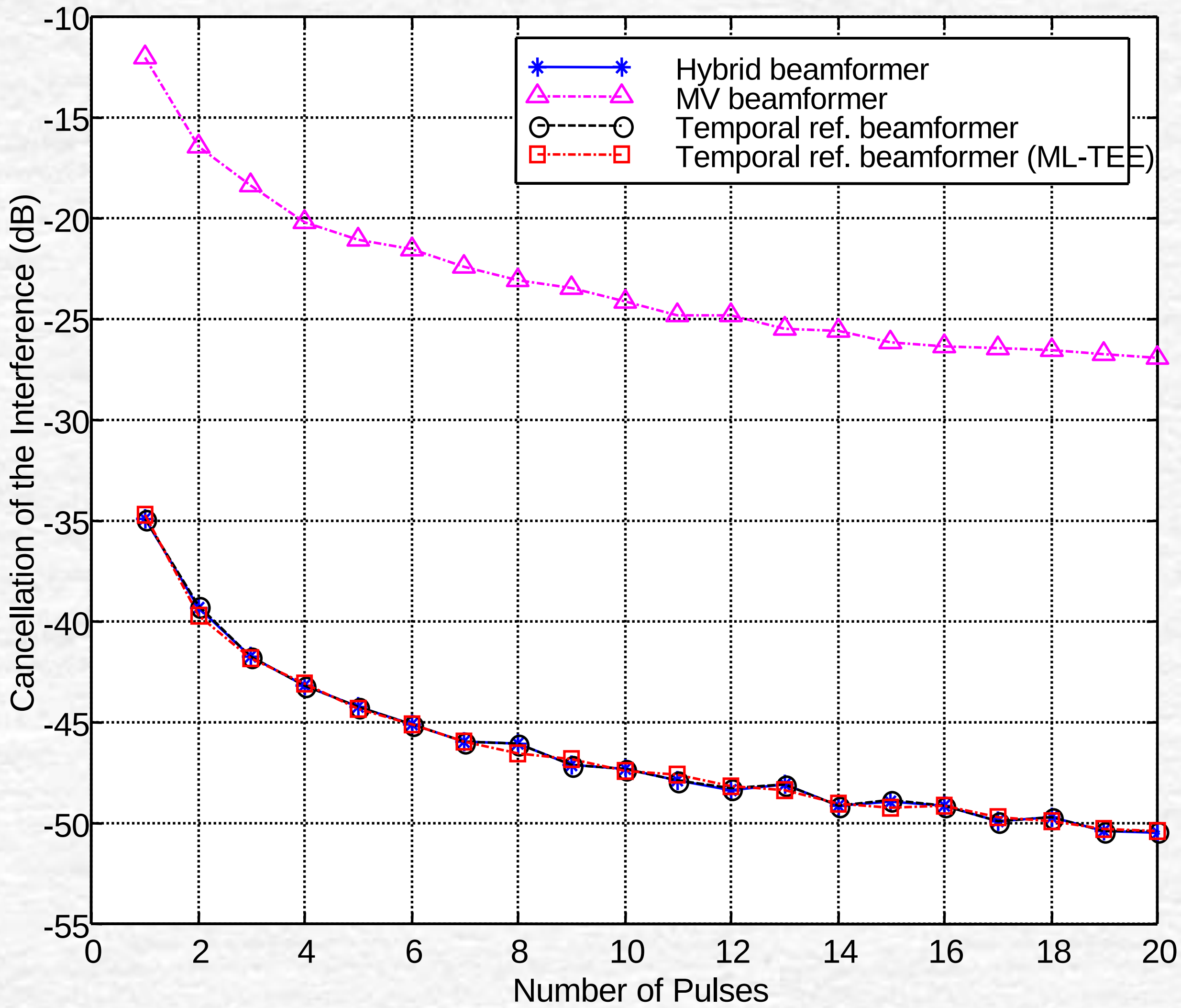
Direct signal + interference



- Although the MVBE tends to the CRB, it has a very poor finite-sample performance.
- The MLE attains the CRB for a small number of samples and does not inherit the poor performance of the MVBE.

SIMULATION RESULTS (2/7)

Direct signal + interference



- The spatial information conveyed by $\mathbf{a}_0$ will be useful when reflections are received.

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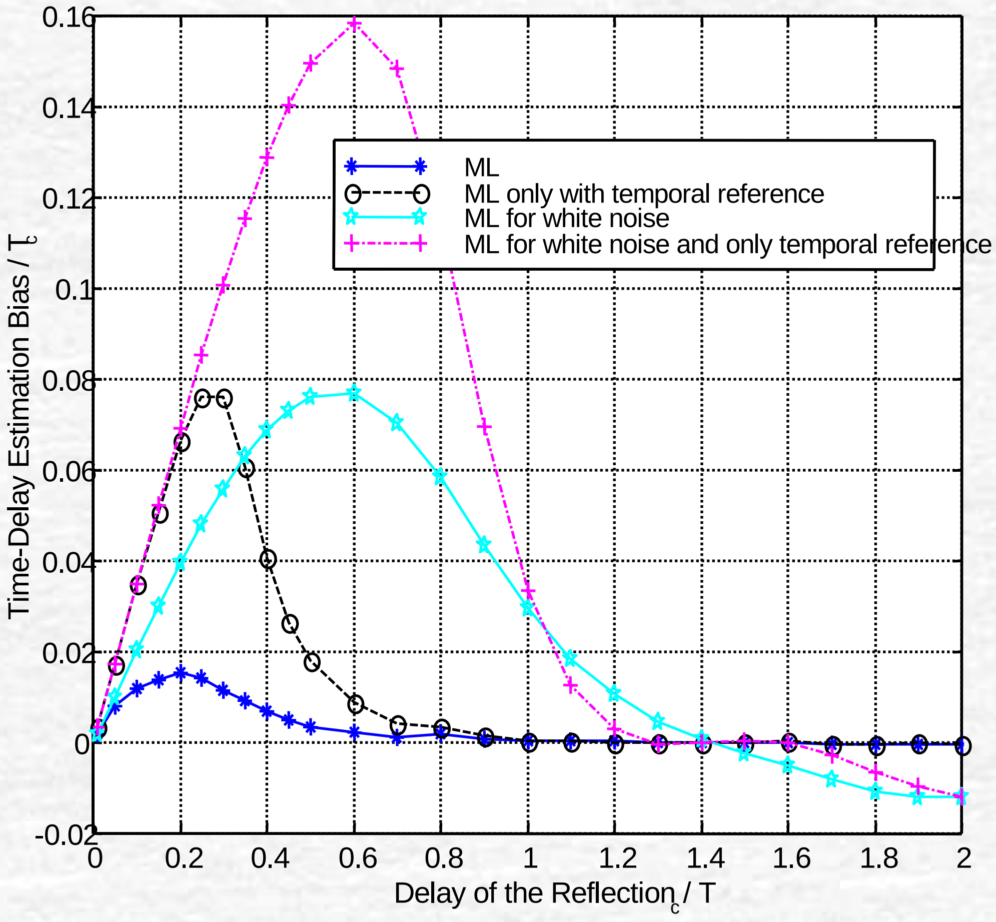
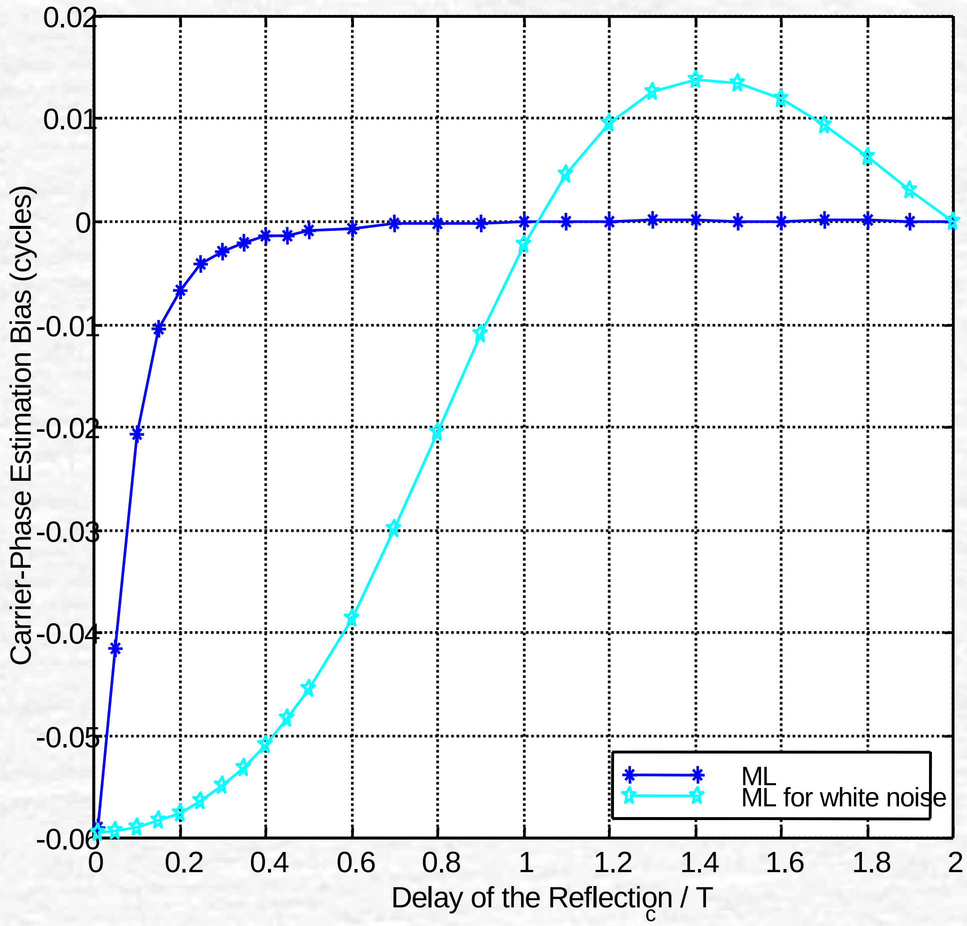
SIMULATION RESULTS (3/7)





- The range of reflection delays that cause a significant bias is reduced by the estimators that assume the noise correlation to be unknown (with respect to their white-noise counterparts).
- The magnitude of the bias is further reduced by the MLE (w.r.t. the ML-TEE).

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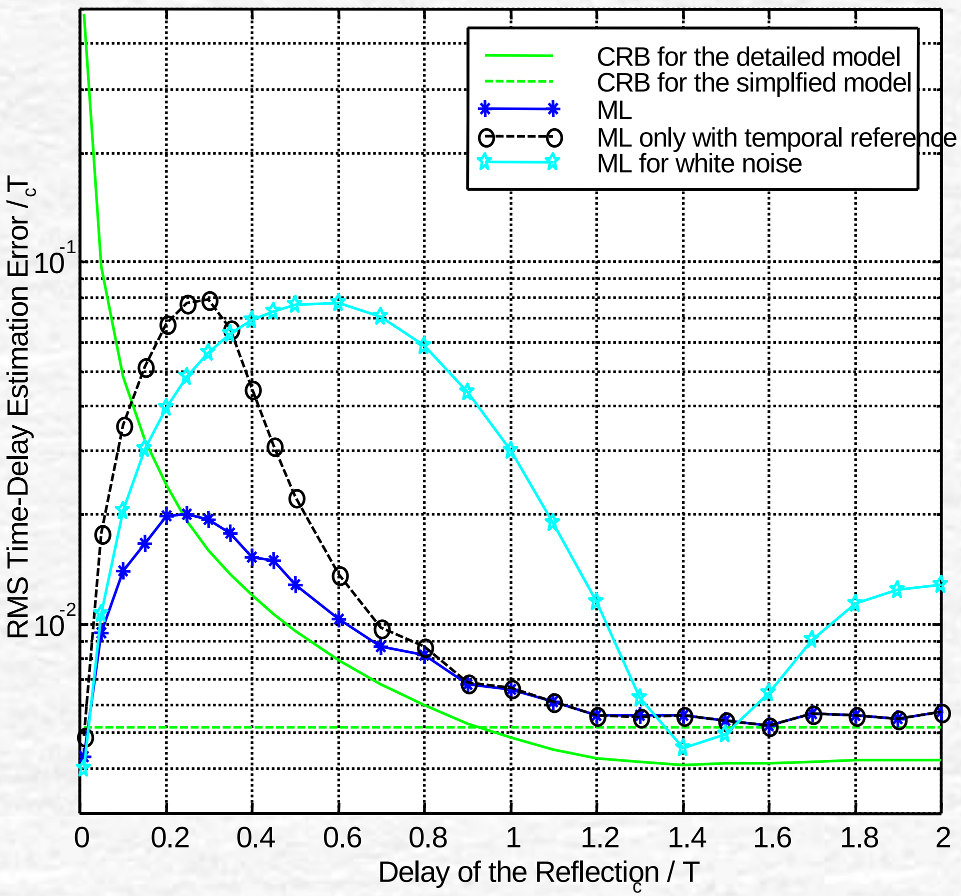
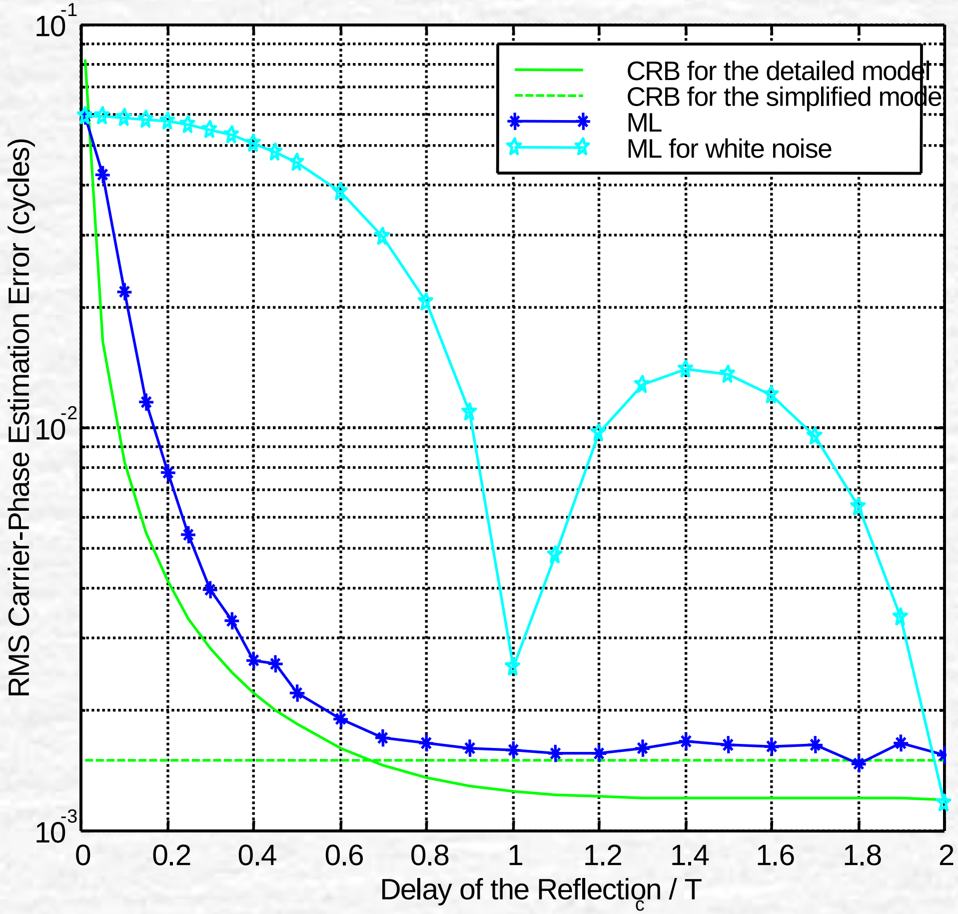
SIMULATION RESULTS (4/7)





- CRB-D < CRB-S for large delay separation and small angular separation.
- MLE approaches the CRB-S for large delays and the CRB-D for small delays.
- By allowing a small bias, the RMSE of the MLE does not tend to infinity as the reflection delay decreases. Advantage with respect to the detailed model.

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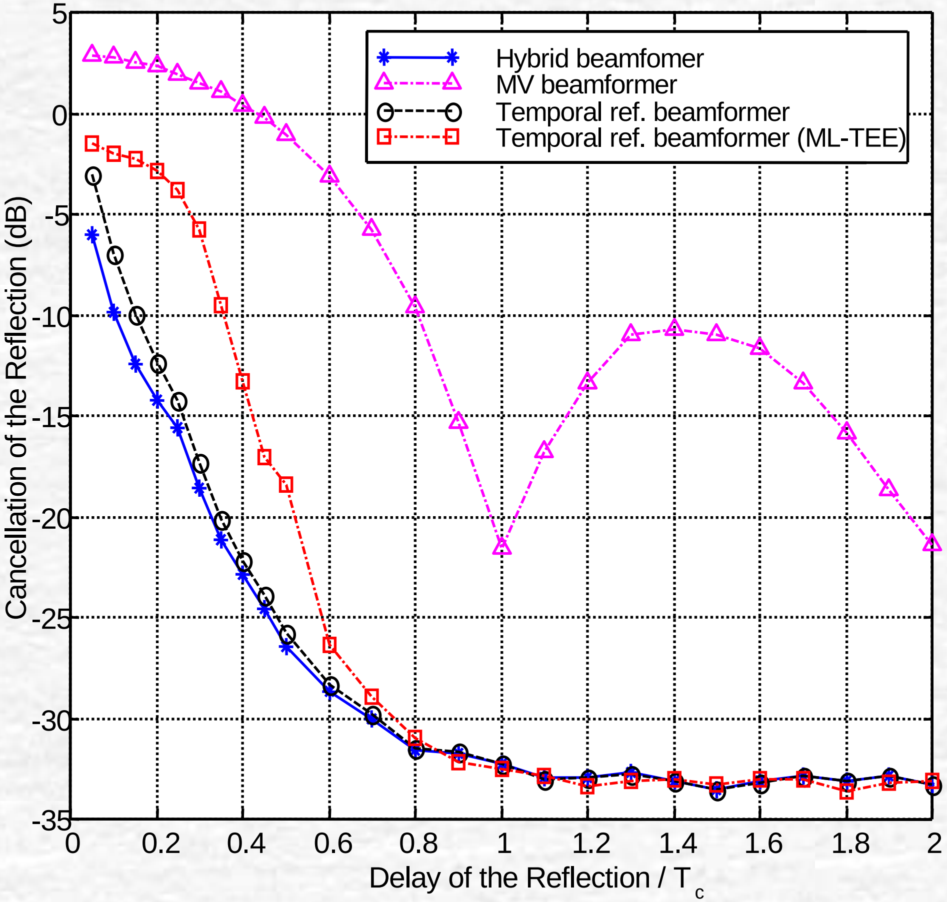
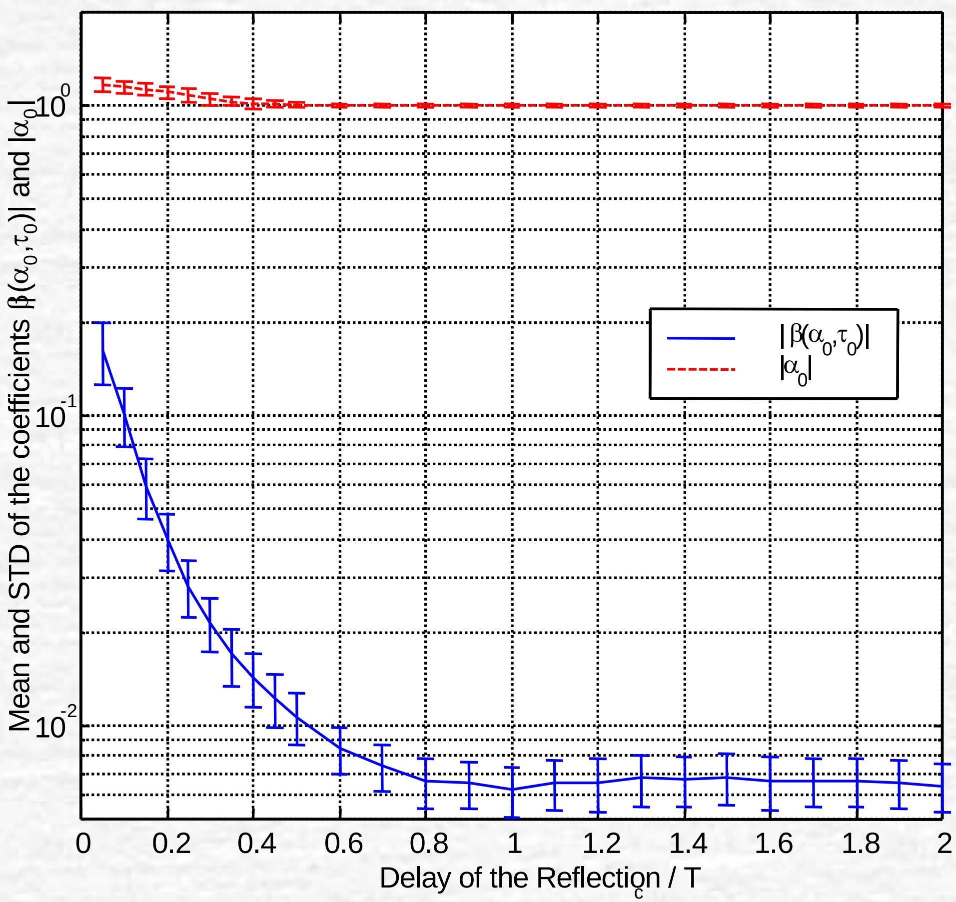
SIMULATION RESULTS (5/7)





- The hybrid beamformer provides the greater attenuation. In the presence of reflections, the knowledge of $\mathbf{a}_0$ is relevant since the HB outperforms the TRB.
- The contribution of the MVB becomes smaller as the delay spacing increases.

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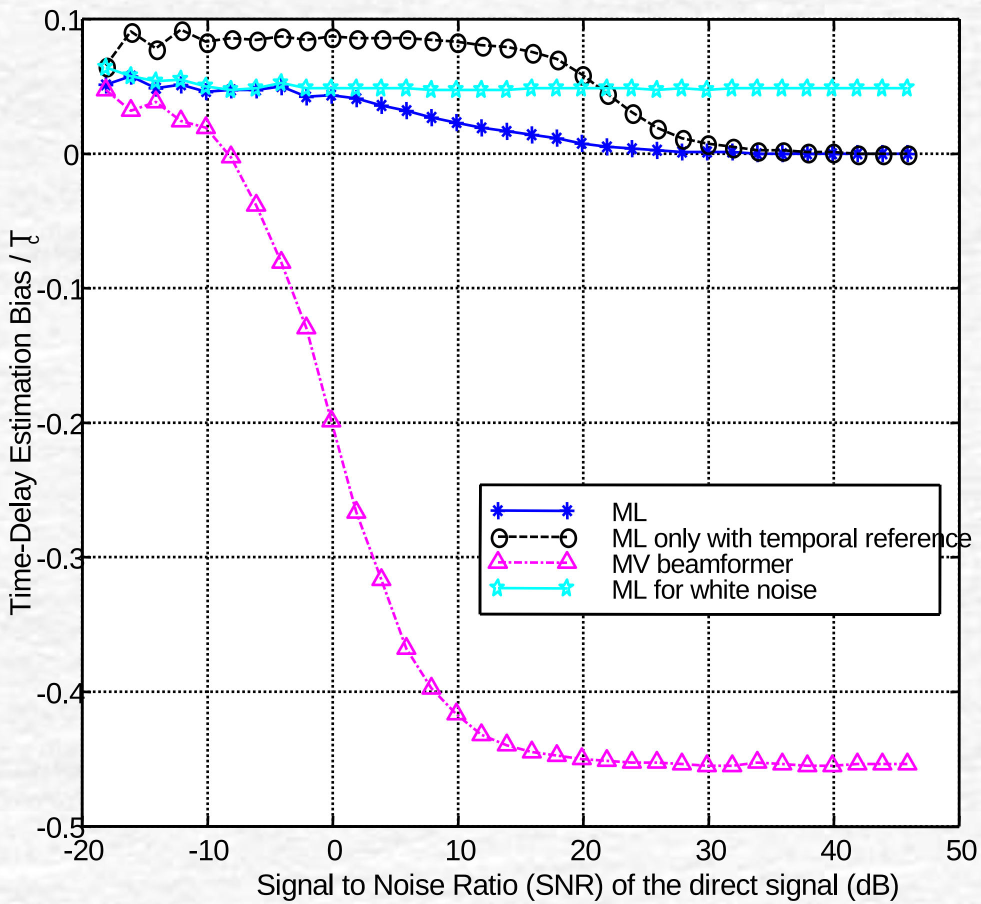
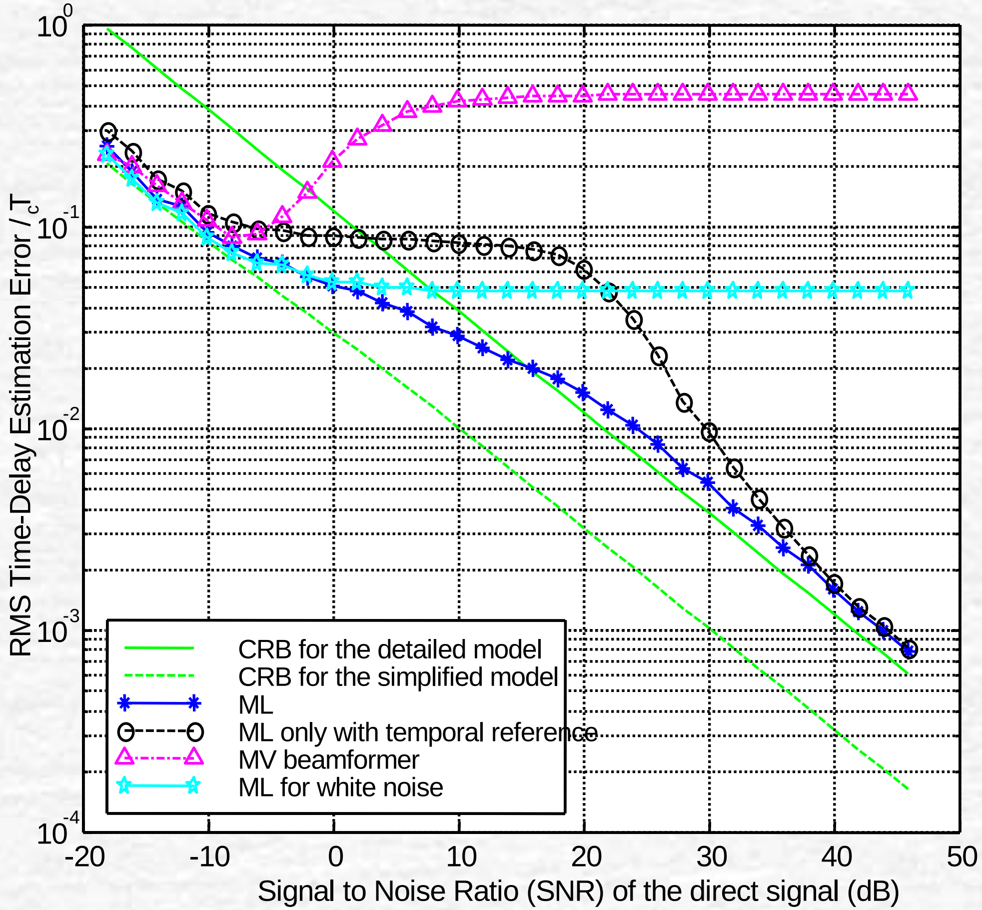
SIMULATION RESULTS (6/7)





- The MLE and the ML-TEE are asymptotically in SNR unbiased. The bias of the MLE starts decreasing at a smaller SNR.
- The RMSE of the MLE starts at the CRB-S and tends to the CRB-D as SNR ↑.

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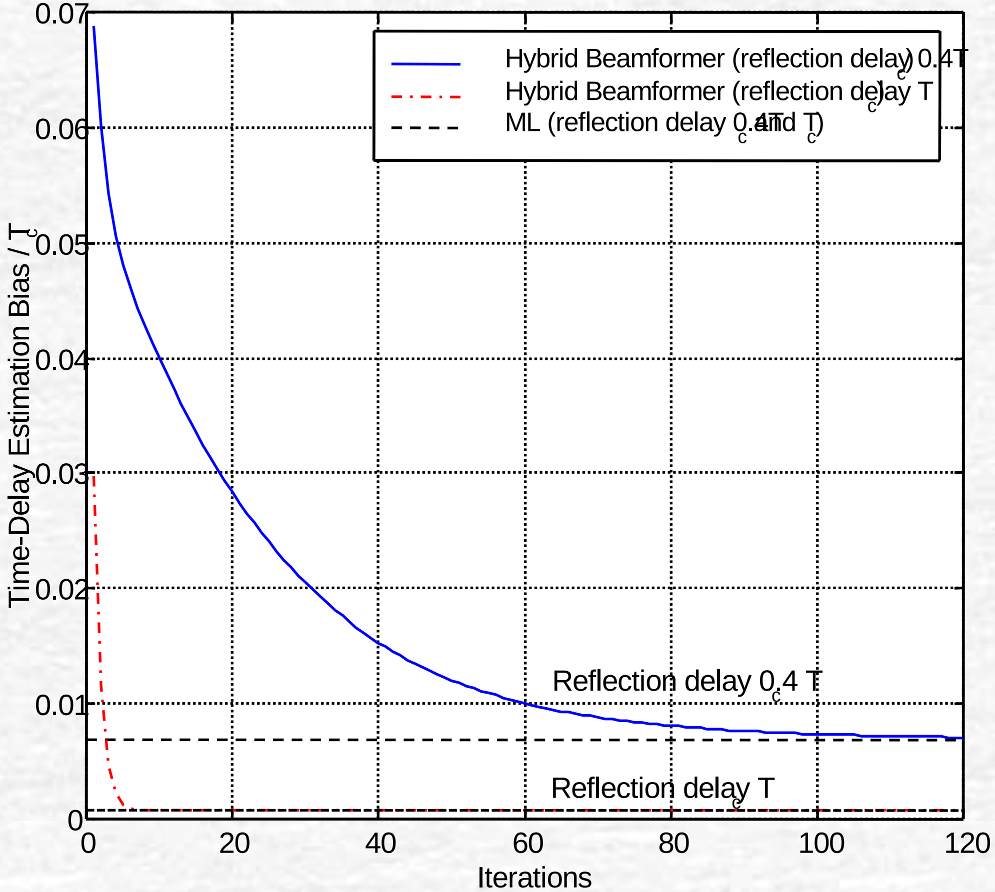
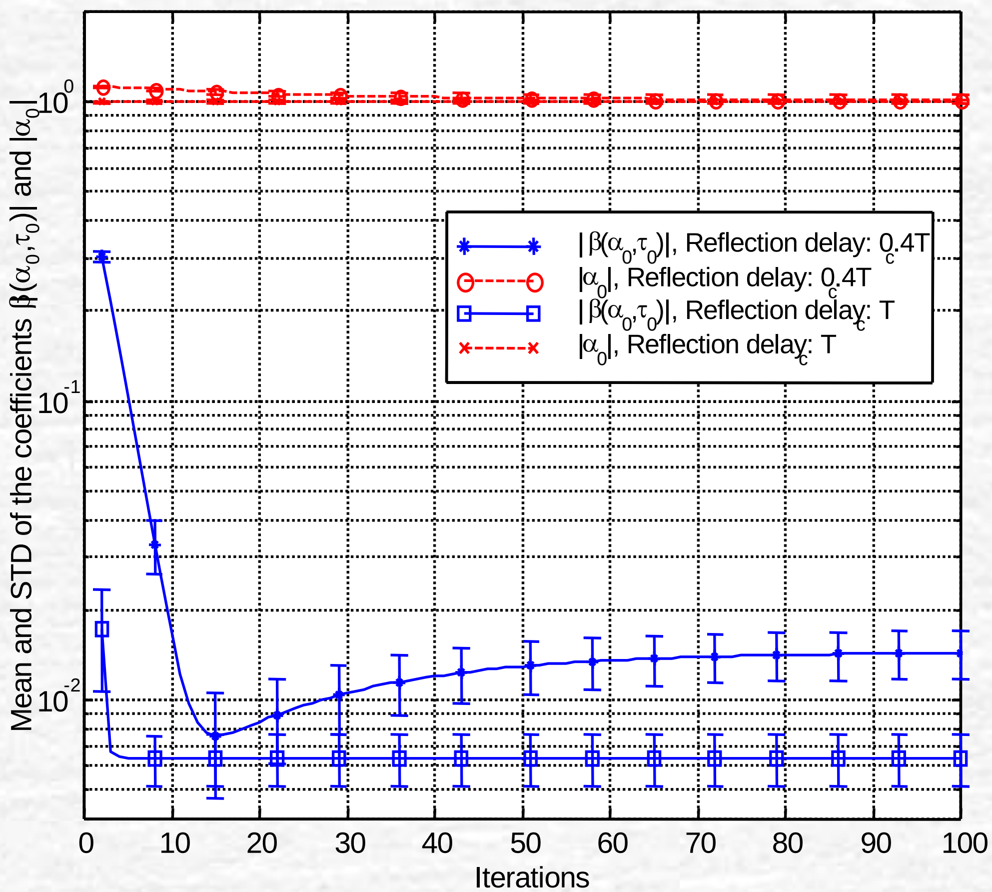
SIMULATION RESULTS (7/7)





- The estimates obtained with the iterative algorithm based on the hybrid beamformer converge to the ML estimates.
- The contribution of the MV beamformer is noticeable in the "acquisition" stage.

EFFECT OF ERRORS ON THE STEERING VECTOR

- ML estimator without spatial reference:

$$\sigma_{TE}^2 = \frac{1}{2\left(\mathbf{d}^*(\tau_0)\mathbf{P}_{\mathbf{s}(\tau_0)}^{\perp}\mathbf{d}(\tau_0)\right)\left(\boldsymbol{\alpha}_0^*\mathbf{Q}^{-1}\boldsymbol{\alpha}_0\right)} \qquad \mathbf{d}(\tau_0) = \frac{d\mathbf{s}(\tau_0)}{d\tau_0}$$

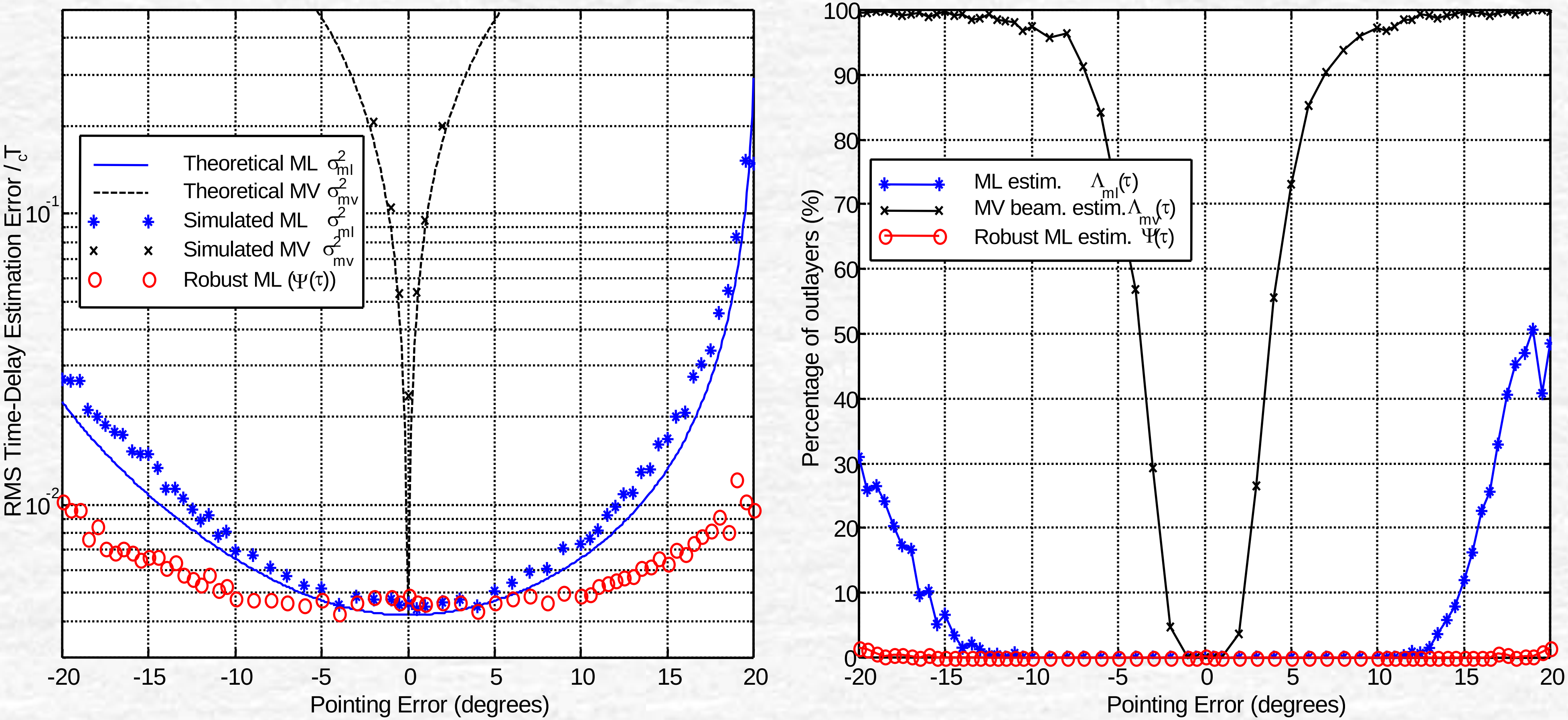
- Minimum variance beamformer:

$$\sigma_{MV}^2 = \frac{\mathbf{a}_0^*\mathbf{R}_{yy}^{-1}\mathbf{Q}\mathbf{R}_{yy}^{-1}\mathbf{a}_0}{2\left(\mathbf{d}^*(\tau_0)\mathbf{P}_{\mathbf{s}(\tau_0)}^{\perp}\mathbf{d}(\tau_0)\right)\left|\mathbf{a}_0^*\mathbf{R}_{yy}^{-1}\boldsymbol{\alpha}_0\right|^2}$$

- ML estimator with spatial reference:

$$\sigma_{ML}^2 = \frac{\mathbf{a}_0^*\mathbf{Q}^{-1}\mathbf{a}_0}{2\left(\mathbf{d}^*(\tau_0)\mathbf{P}_{\mathbf{s}(\tau_0)}^{\perp}\mathbf{d}(\tau_0)\right)\left|\mathbf{a}_0^*\mathbf{Q}^{-1}\mathbf{a}_0\right|^2}$$

SIMULATION RESULTS



- Good agreement between theoretical and simulated values.
- The ML allows errors up to 10º (≈beamwidth at 3dB). The robust estimator widens (in about 5º) the range of those tolerable errors.

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ADDRESSED PROBLEM

- Code synchronization of a desired user in a DS-CDMA communications system is an essential task
  - Multiple-access interference in a near-far environment usually necessitates the use of multiuser detectors.
  - Most multiuser detectors require precise knowledge of several parameters (timings, phases and powers) of the received signals.
  - Some parameters, like the spatial signature, are easily obtained once the code-timing is acquired.
  - Moreover, the process of code synchronization may limit the capacity of a DS-CDMA system.

OBJECTIVES, PROPOSED SOLUTION

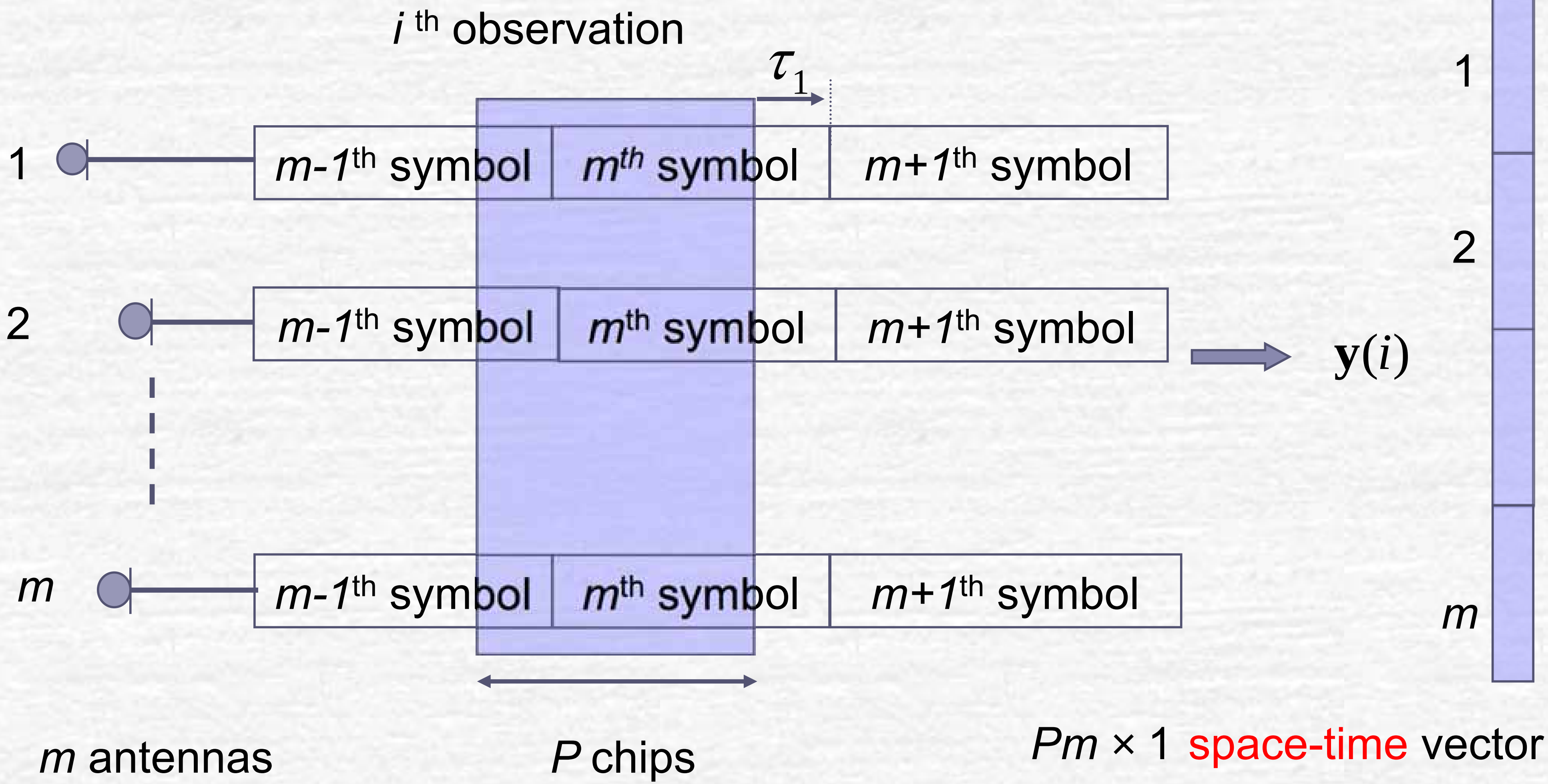
- Exploitation of the whole structure (spatial: provided by the array, temporal: provided by the codes) of the signals.
- Cancellation of possible external interference.

- Near-far, frequency-nonselective, slowly fading channel.
- Knowledge of the symbols of the desired user is required ⇒ training sequence (acquisition), decisions' feedback (tracking).

- Near-far resistant, single-user technique (vs. joint estimation).
  - Joint estimation of the parameters of all users leads to centralized and excessively complex solutions.

SPACE-TIME VECTORS (1/2)

Asynchronous DS-CDMA system



SPACE-TIME VECTORS (2/2)

$$\underbrace{\mathbf{y}(i) = \left[\boldsymbol{\alpha} \otimes \mathbf{A}(\tau_1)\right] \mathbf{d}_1(i)}_{\text{desired user's contribution}} + \underbrace{\mathbf{e}(i)}_{} \qquad i = 0 \ldots M-1$$

• desired user´s contribution

• multiple-access interference
• external interference
• noise

$\boldsymbol{\alpha}$: spatial signature (fading coefficients). UNKNOWN.

$\mathbf{A}(\tau_1)$: “First part” and “last part” of the shifted PN sequence.

$\mathbf{d}_1(i) = \left[d_1(i) \;\; d_1(i-1)\right]^T$: desired user’s training symbols.

INTERFERENCE MODELING

SPACE-TIME COVARIANCE

$$\mathrm{E}\left\{\mathbf{e}(i)\mathbf{e}^{*}(l)\right\} = \mathbf{Q}\,\delta_{i,l}$$

- The estimator will be able to attenuate any interfering signal that exhibits a certain space and/or time structure.

- Each antenna adds *P* degrees of freedom.
- Each user occupies only 2 degrees of freedom, and each point interferer occupies between 1 and *P* degrees.

Previous works can be viewed as particular cases of the technique in this paper:

$$\mathbf{Q} = \mathbf{Q}_{sp} \otimes \mathbf{I}_{P}$$

Only the “spatial diversity” is exploited. It assumes that the signals are temporally white.

$$\mathbf{Q} = \mathbf{I}_{m} \otimes \mathbf{Q}_{te}$$

Only the “temporal diversity” is exploited. It assumes that the signals do not present definite spatial signatures.

ML ESTIMATOR

1) Closed-form estimation of the correlation matrix **Q**.

2) Asymptotically efficient (large sample) approximation of the determinant

$$\ln\left|\mathbf{I}+\mathbf{B}\right| \approx \mathrm{Tr}\left\{\mathbf{B}\right\}, \qquad \mathbf{B}\rightarrow\mathbf{0}$$

3) Closed-form estimation of the spatial signature α .

1D code-timing estimator

$$\hat{\tau}_1 = \arg\max_{\tau_1} f(\tau_1)$$

Unstructured space-time correlation estimate

$$\hat{\mathbf{W}} = \hat{\mathbf{R}}_{yy} - \hat{\mathbf{R}}_{yd}\hat{\mathbf{R}}_{dd}^{-1}\hat{\mathbf{R}}_{yd}^{*}$$

CORRELATION MATRIX ESTIMATE (1/2)

- Reduction of the duration of the observation period or length of the training sequence *M*.

- Bound on *M*: $M \geq mP+2$ ⇨ $\hat{\mathbf{W}}$ is non-singular with probability 1.
- A non-singular estimate $\hat{\mathbf{W}}_s$ can be obtained from $\hat{\mathbf{W}}$ if *M* is greater than the dimension of the interference subspace of $\mathbf{Q}$, although $M < mP+2$.

- $\hat{\mathbf{W}}_s$ is obtained as the matrix with an "interference and noise subspaces structure" that is closest to $\hat{\mathbf{W}}$ in the sense of the Frobenius norm:

$$\hat{\mathbf{W}}_s = \mathbf{P} + \sigma^2 \mathbf{I}$$

$$\min_{\mathbf{P},\sigma} \left\| \hat{\mathbf{W}} - \mathbf{P} - \sigma^2 \mathbf{I} \right\|_F^2 \quad \text{subject to} \quad \operatorname{rank}\{\mathbf{P}\} = d$$

CORRELATION MATRIX ESTIMATE (2/2)

- Other alternatives
  - Diagonal loading $\hat{\mathbf{W}}_d = \hat{\mathbf{W}} + \lambda \mathbf{I}$

    Simpler, but the structured estimate performs slightly better.

  - Pseudo-inverse $\hat{\mathbf{W}}^{\#}$

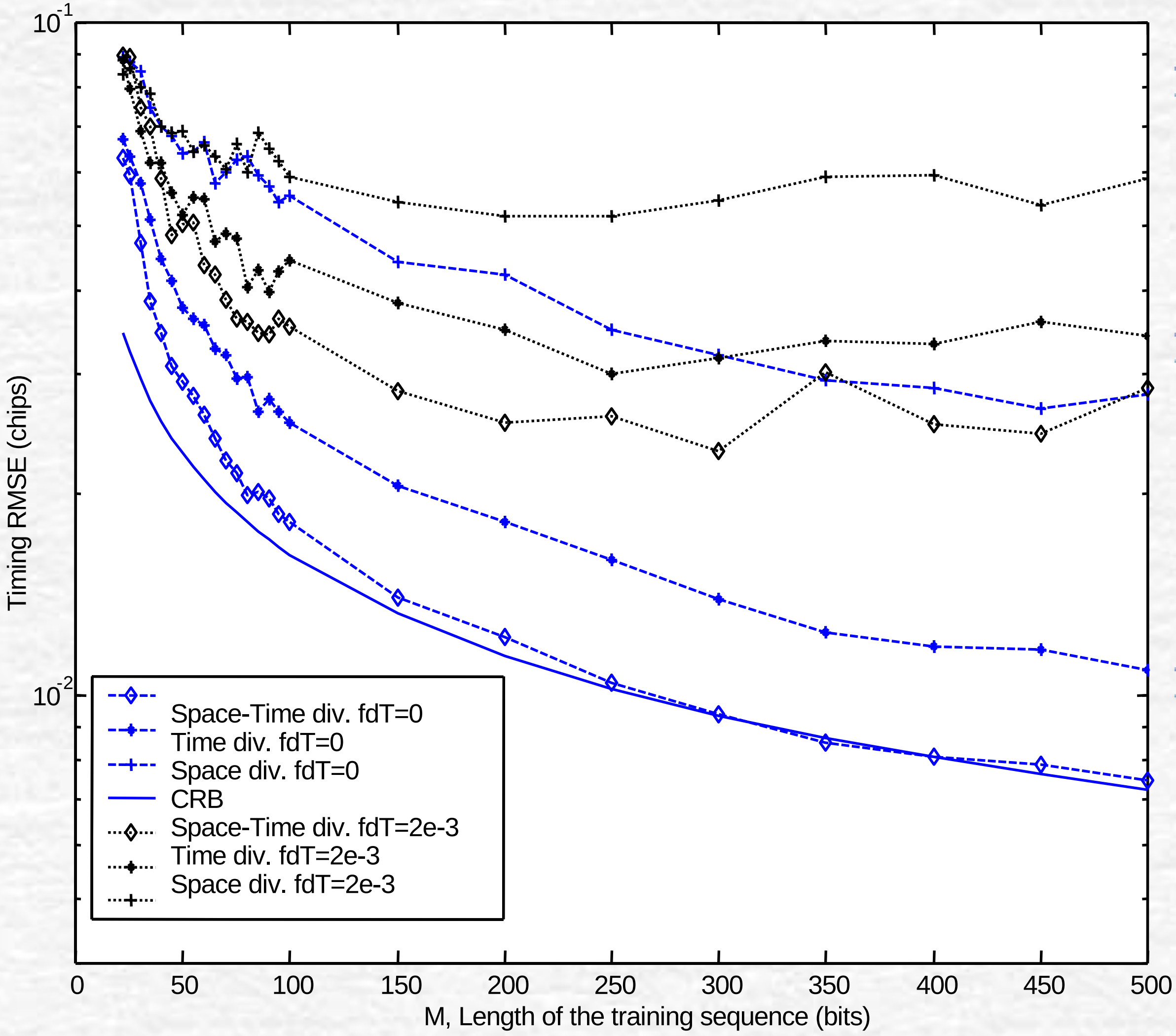
    It fails because it neglects the projection of the data onto the noise subspace, instead of emphasizing it.

SIMULATION PARAMETERS

- Comparison of our estimator (space-time diversity, **STD**) with other two methods that also employ antenna arrays, denoted as space-diversity (**SD**) and time-diversity (**TD**) estimator.
- BPSK, N=15 chips/bit Gold sequence, *m*=4 antennas spaced $\lambda/2$ apart.
- *M*=80 bits, EbNo=4 dB per antenna (total: 10 dB).
- Near-far ratio (NFR): 10 dB.

- Two types of channels:
  - "Static" channel
  - Mobile channel.
    - Angular spread 5º
    - Doppler spread $f_d T = 2 \cdot 10^{-3}$. (900 MHz carrier frequency, 50 kb/s data rate and 120 km/h speed)

SIMULATION RESULTS (1/4)



- STD outperforms TD and SD in the static and mobile channels.
- STD attains the CRB in the static channel. All methods are deteriorated in the mobile channel.
- In this scenario, the SD is the worst estimator since it is the method with the smallest number of degrees of freedom.

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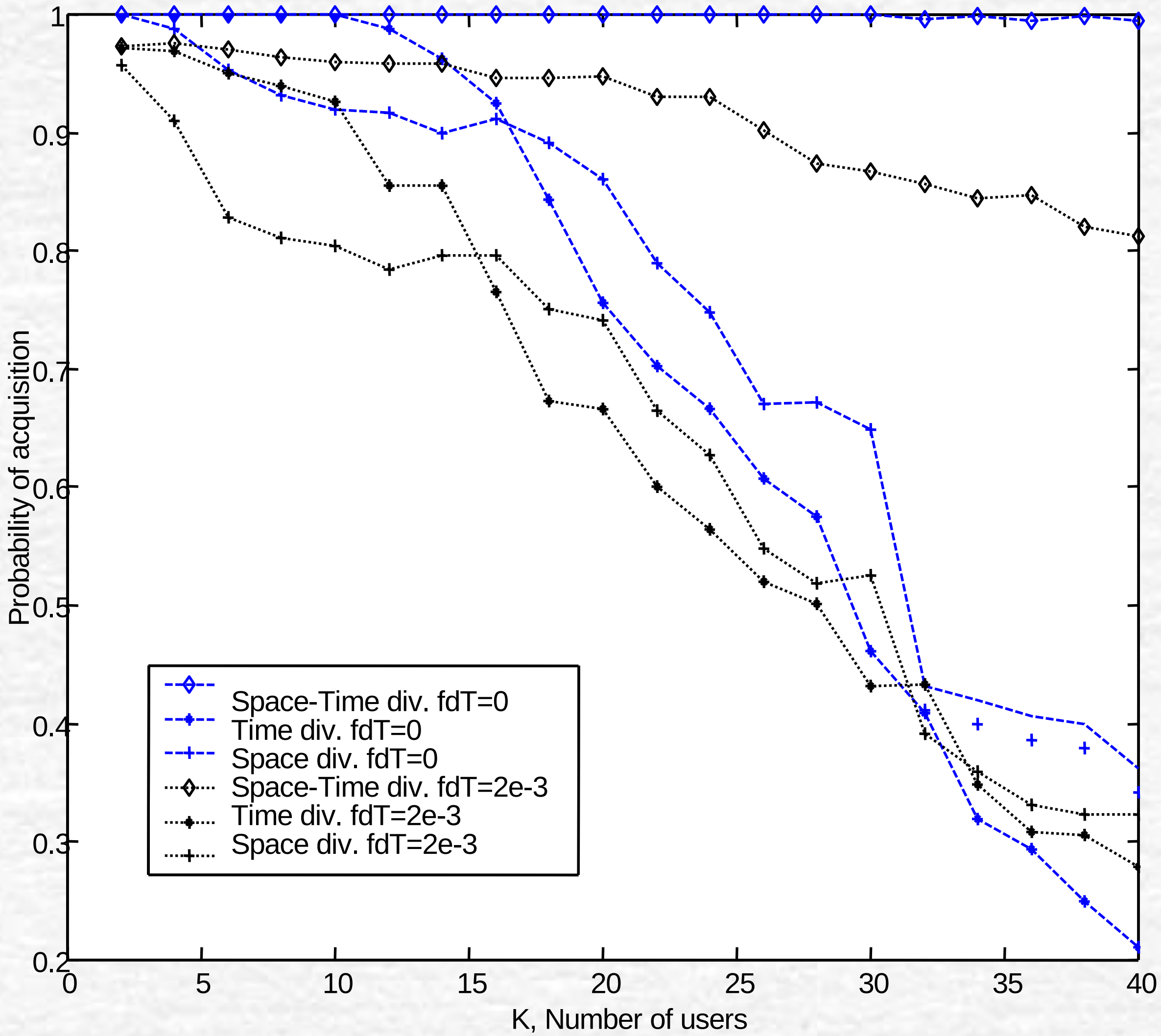
SIMULATION RESULTS (2/4)



- Only with the STD estimator it is possible to have more users than the spreading gain, i.e., *K>P*.
- SD and TD experience a serious deterioration when the number of users exceds the length of the code, and completely fail when *K>2P*.
- Using the STD the number of users may be increased beyond twice the code length. *K*=40 users can be allocated using a *P*=15 spreading gain.

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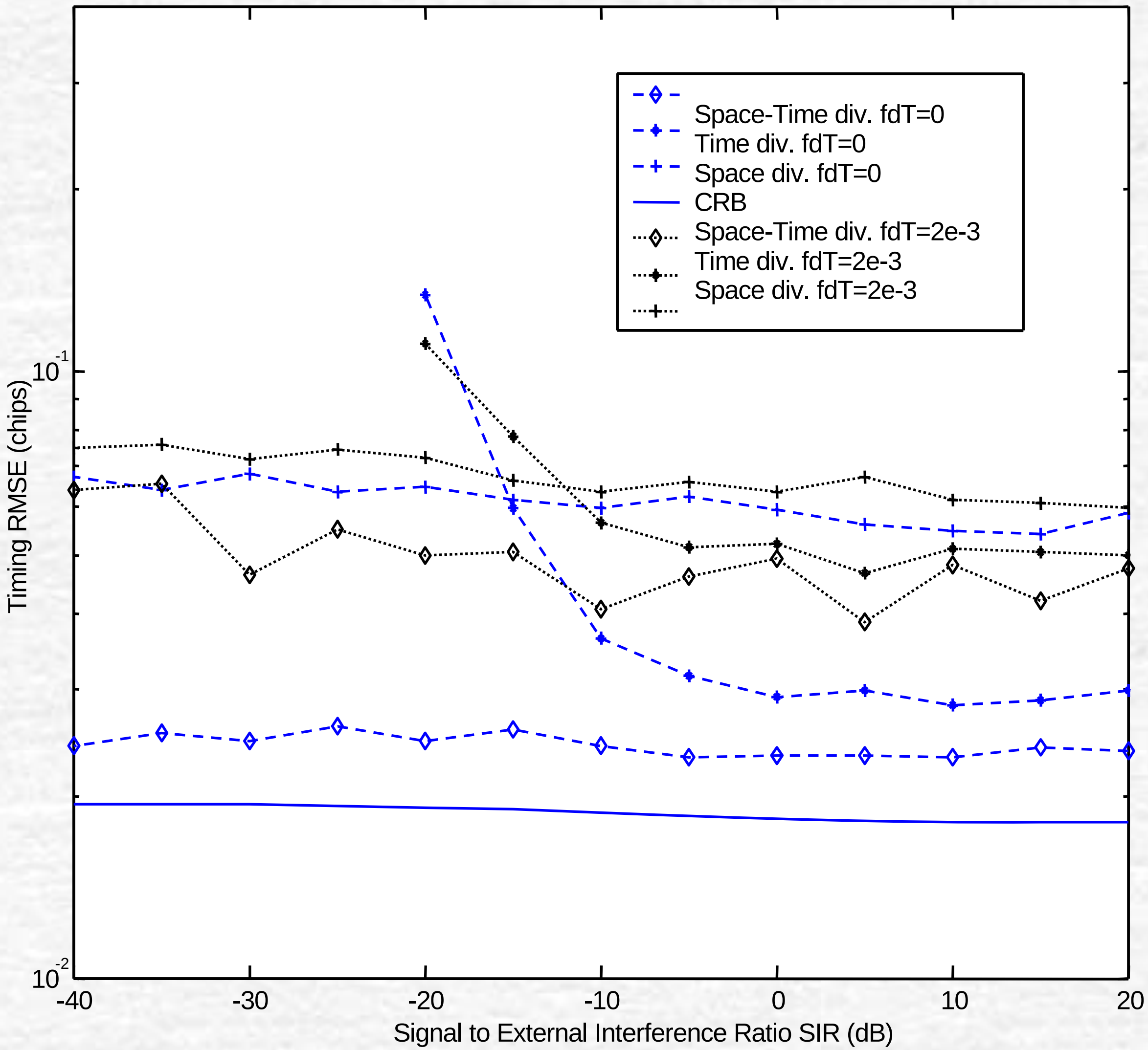
SIMULATION RESULTS (3/4)



- The STD and SD estimators present satisfactory performance when the SIR is low, such as SIR < -15dB.
- Their performance is nearly insensitive to the SIR except for extremely low SIR in the mobile channel.
- Nevertheless, the STD always outperforms the SD, and the SD is not near-far resistant.

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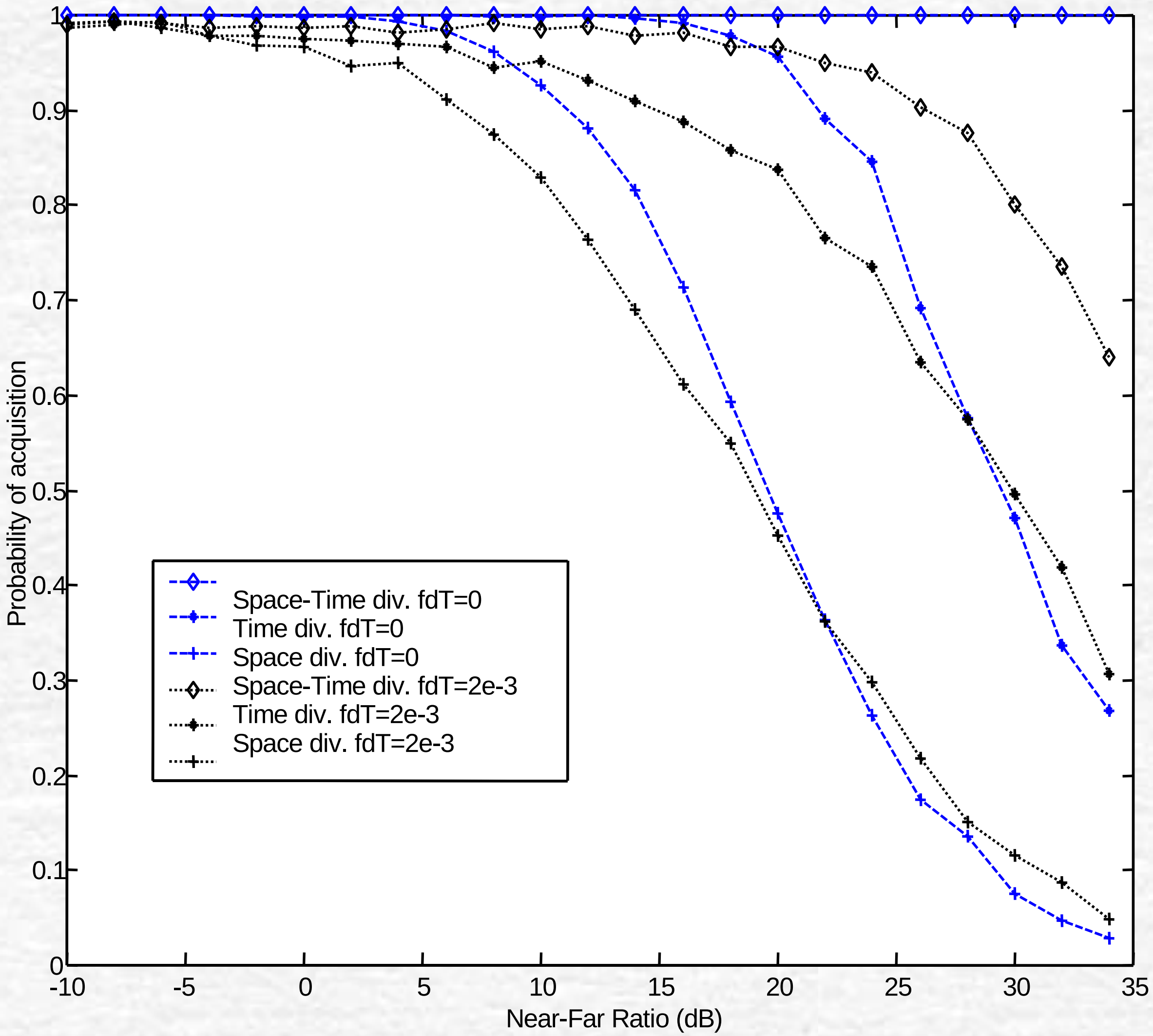
SIMULATION RESULTS (4/4)



- The STD estimator performs satisfactorily in the mobile channel up to a NFR equal to 35dB.
- This represents an improvement of 8dB and 18dB with respect to the TD and SD estimators, respectively.

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CONCLUSIONS

- General time-delay estimation problem in spatially colored noise.
  - Derivation of an asymptotically efficient estimator that makes possible the use of the IQML algorithm.
- Estimation of the time delay and the carrier phase using the knowledge of the steering vector of the direct signal.
  - Connection with hybrid beamforming.
  - Analysis of the effect of errors in the steering vector.
- Code synchronization in DS-CDMA communications systems.
  - Exploitation of the spatial and temporal structure of the signals.

TOPICS FOR FUTURE RESEARCH

- Application of the ML principle with correlation matrices that have a certain structure.
- Use of unstructured stochastic spatial signatures.
- Introduction of the temporal correlation using a parametric model.
- Deeper exploration of the bias-RMSE trade-off.

SIGNAL STRUCTURE: GALILEO

- Chip rates: 3.069, 15.345 and 24,552 Mchips/s
- Data rates: 1.5, 3 and 24 kbps
- Complex spreading, coding and interleaving
- Chip-shaping: Square root raised cosine pulse with roll-off = 0.2
- 3 carrier frequencies chosen among 1589.742, 1561.098, 1256.244, 1598.949, 1250.106 and 5014.746 MHz
- Some PN sequences may not carry data modulation.
- Target UERE on the order of 1-2 meters with single carrier ranging and 1-2 cm with three-carrier differential phase positioning techniques.
- Adoption of a reginal IGSO / GEO or, alternatively, a global MEO satellite constellation.

CONVENTIONAL RECEPTION SCHEMES

- Received signal: $$x(t) = \alpha_0 \sum_l d(l)\, p(t - lT) + w(t)$$

ML principle $\Downarrow$ $w(t):\ N(0, \sigma^2)$

- Costas loop: $$\hat{\varphi}_0 = \angle y(lT + \hat{\tau}_0) d^*(l)$$

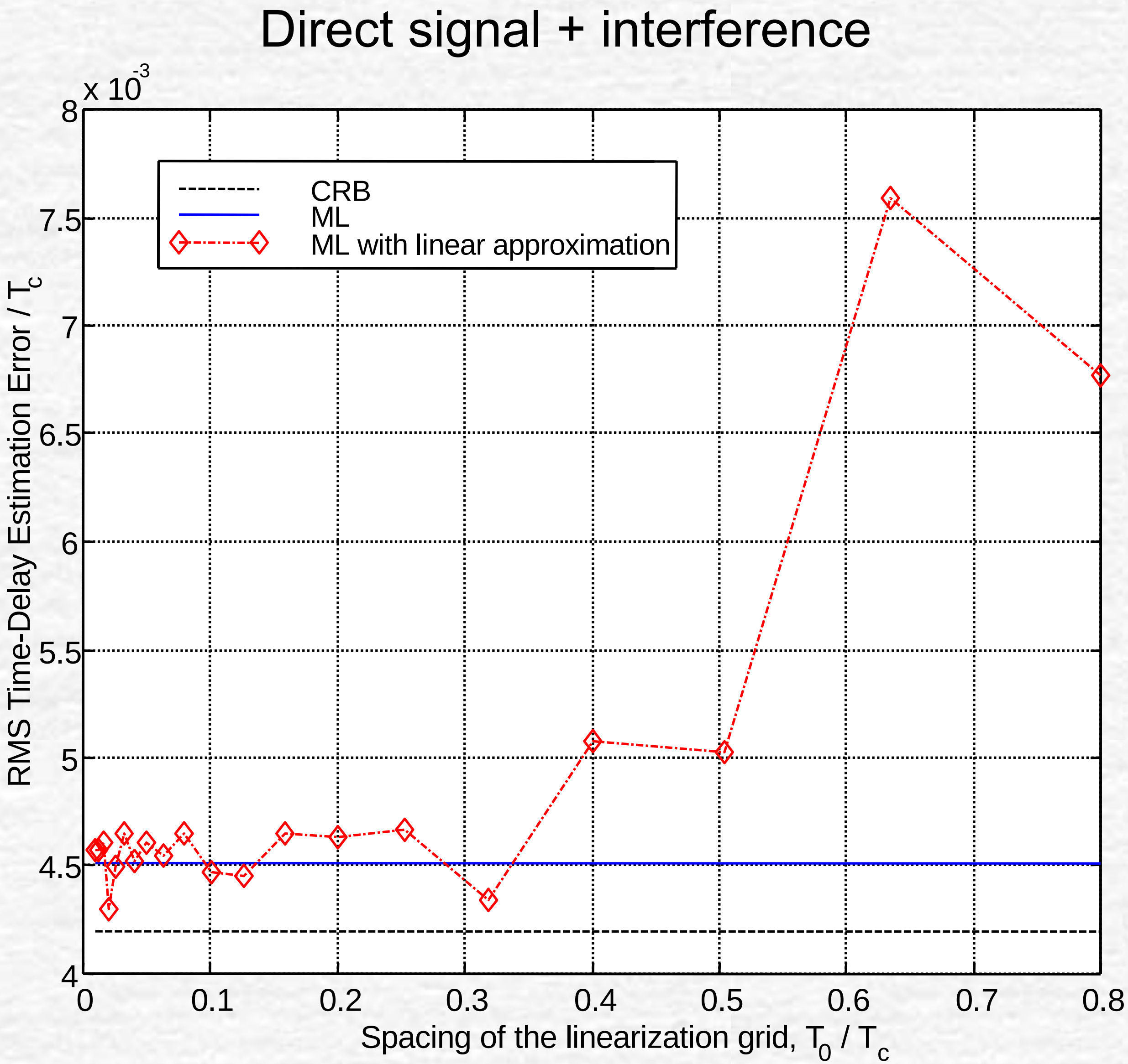
- Decision and phase directed. Coherent. $$L_c(\tau_0) = \mathrm{Re}\left\{ \sum_l \hat{d}^*(l)\, e^{-j\hat{\varphi}_0}\, y(lT + \tau_0) \right\}$$

- Non-data aided, phase independent. Non coherent. $$L_{nc}(\tau_0) = \sum_l \left| y(lT + \tau_0) \right|^2$$

(Decision and phase directed / Non-data aided: DLL)

y(t): despreader output

SIMULATION RESULTS (3/8)



- Good performance despide the round shape of the Nyquist pulses.

EFFECT OF ERRORS ON THE STEERING VECTOR (1/2)

- The asymptotic variance of the time-delay estimates is computed as

$$\sigma_\tau^2 = \frac{1}{N} \frac{\lim_{N\to\infty} N \mathrm{E}\left\{\left(\Lambda'(\tau_0)\right)^2\right\}}{\left(\lim_{N\to\infty} \Lambda''(\tau_0)\right)^2}$$

- ML estimator: $\Lambda_{ML}(\tau_0) = \dfrac{\left|\mathbf{a}_0^* \hat{\mathbf{R}}_{yy}^{-1} \hat{\mathbf{r}}_{ys}(\tau_0)\right|^2}{\hat{P}_s - \hat{\mathbf{r}}_{ys}^*(\tau_0) \hat{\mathbf{R}}_{yy}^{-1} \hat{\mathbf{r}}_{ys}(\tau_0)} = \dfrac{\Lambda_{MV}(\tau_0)}{1 - \Lambda_{TE}(\tau_0)}$

- $\boldsymbol{\alpha}_0$ = actual spatial signature.
- $\mathbf{a}_0$ = nominal or a priori steering vector.

- The theoretical results are obtained by assuming the absence of reflections.

ROBUST ESTIMATOR

- The steering vector is parameterized by a nuisance/calibration parameter.

$$\mathbf{a}_0 = \mathbf{a}(\rho)$$

- Robust method vs. autocalibration method.

- Robust estimator $$\Psi(\tau_0) = \mathrm{E}_\rho\left\{\Lambda_{ML}(\tau_0;\rho)\right\}$$

$$\Lambda(\tau_0;\rho) = \frac{\mathbf{a}^*(\rho)\hat{\mathbf{W}}(\tau_0)\mathbf{a}(\rho)}{\mathbf{a}^*(\rho)\hat{\mathbf{R}}_{yy}^{-1}\mathbf{a}(\rho)}$$

- Approximations: $\mathbf{a}(\rho) \approx \mathbf{a}_0 + \rho\mathbf{b}_0 + \frac{\rho^2}{2}\mathbf{h}_0$ $\qquad \frac{1}{1+x} \approx 1 - x + x^2$